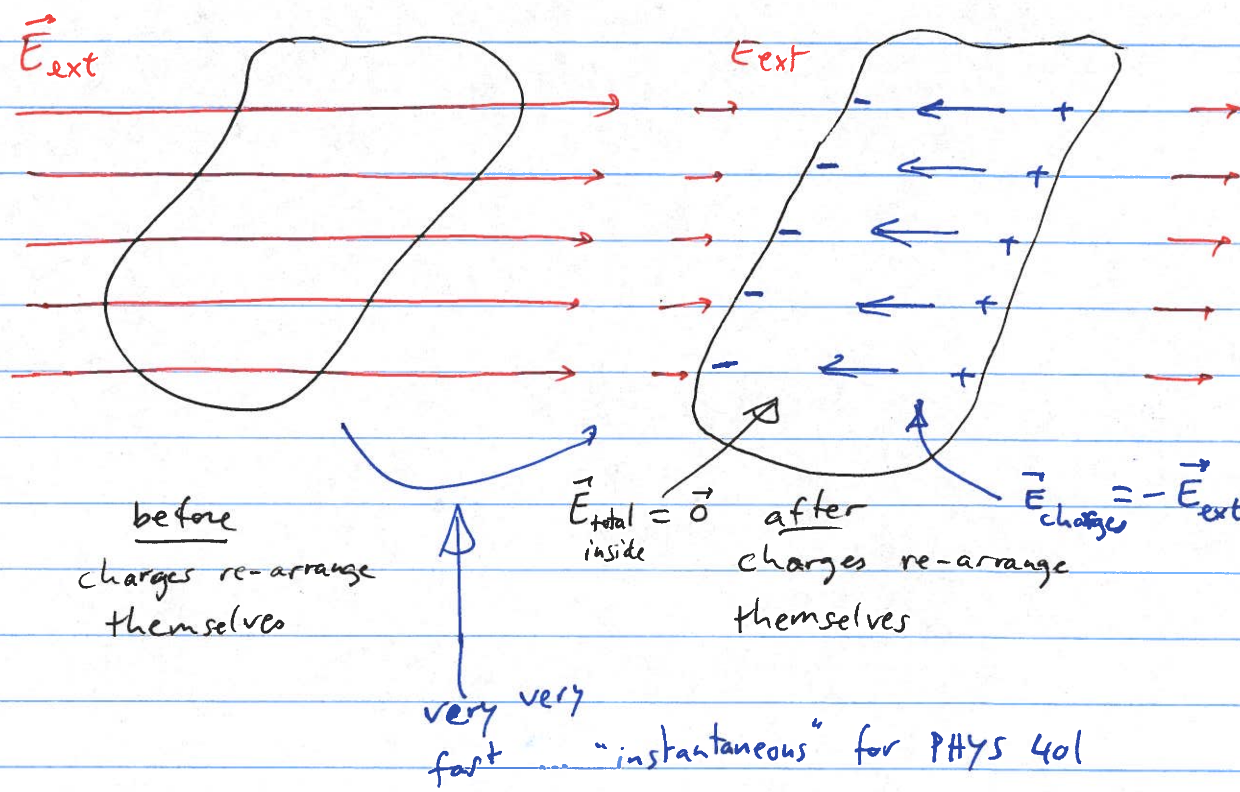
Wednesday, February 22, 2023

"Perfect conductors" (e.g. metals such as Cu, Al, Au, Ar, etc)

definition: A body with an unlimited supply of free charges (positive and negative) that are free to move around within its volume and on its surface.

property #1: Inside a conductor $\vec{E} = 0$



$\Rightarrow$ $\vec{E}_{inside\ conductor} = \vec{0}$ [if it wasn't zero, then the free charges would be pushed by $\vec{E}_{inside}$ and would thus re-arrange themselves]

$\Rightarrow$ $V =$ constant inside conductor $\rightarrow$ conductor is an equipotential.

Property #2 : Inside a conductor $\rho(\vec{r}) = 0$

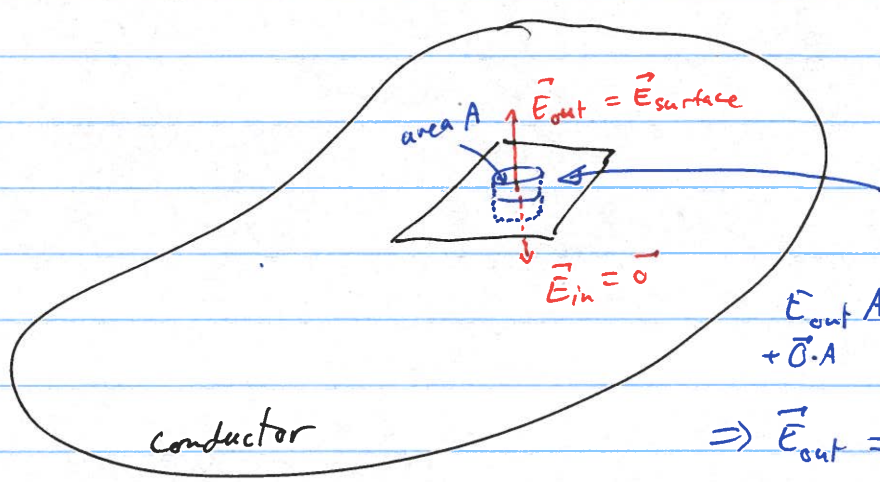
$$\vec{\nabla} \cdot \vec{E}_{inside} = 0 = \frac{\rho_{inside}(\vec{r})}{\varepsilon_0} \implies \boxed{\rho_{inside}(\vec{r}) = 0}$$

$\implies$ **Property #3 :** All charges reside on the surface of the conductor

[includes charges from conductor + any added free charges]

Property #4 : The E-field just outside the conductor is perpendicular to its surface with $\boxed{\vec{E}_{surface} = \frac{\sigma}{\varepsilon_0}\hat{n}}$

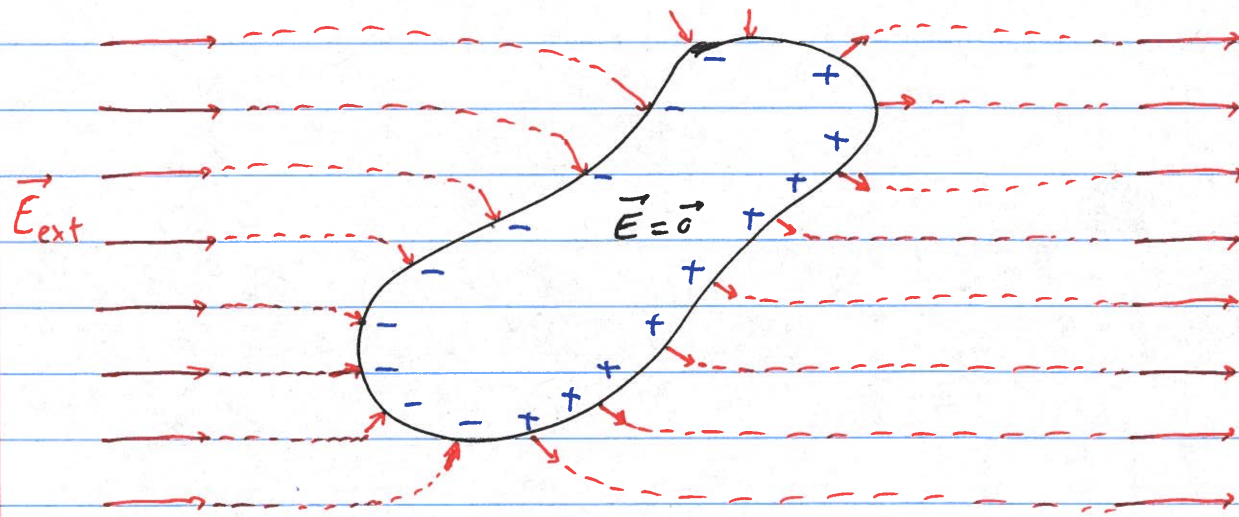
note: If $\vec{E}_{surface}$ had a parallel component, then charge would flow and redistribute itself.



$$E_{out}A + \vec{0} \cdot A = \frac{1}{\varepsilon_0}\sigma A$$

$$\implies \vec{E}_{out} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

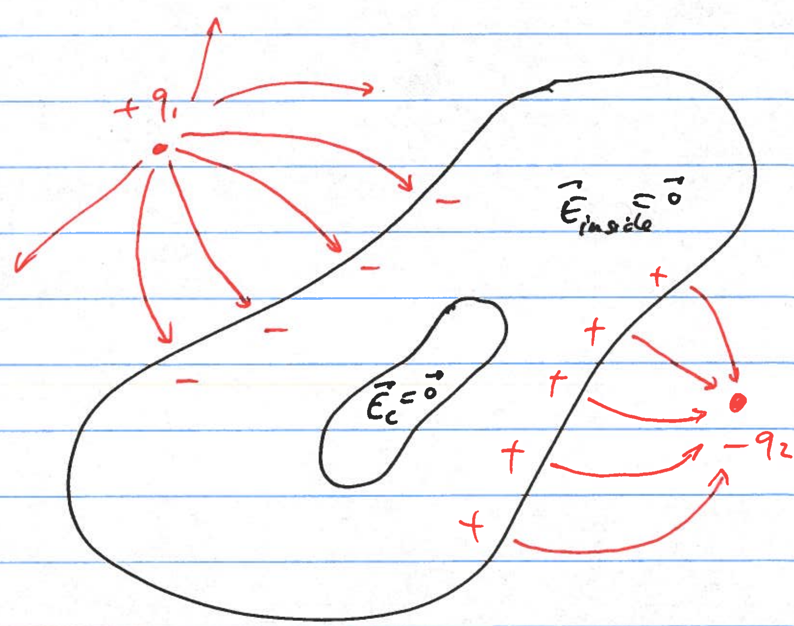
Illustration: Conductor in an external E-field



Screening and shielding

Case 1: Consider a conducting volume $\mathbb{V}$ with an empty cavity $\mathbb{V}_c$ inside of it.

↳ $\vec{E}_c$ inside the cavity is zero for any arrangements of charges / fields outside of $\mathbb{V}$. (and $\mathbb{V}_c$)



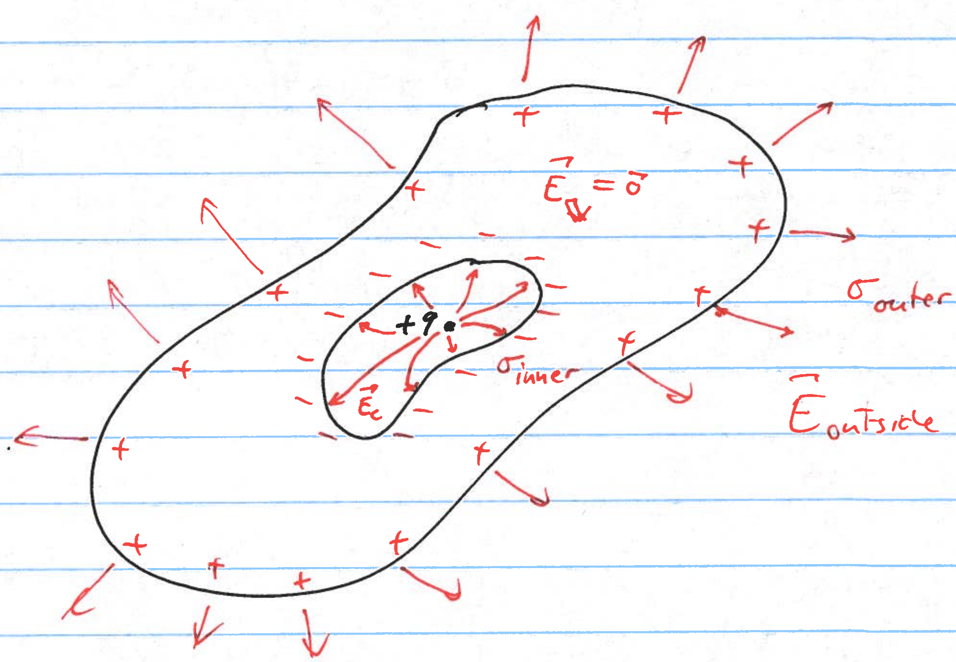
note: q_1 & q_2 are attracted to the conductor

<u>Physics proof:</u> Consider the conductor without the cavity, then $\vec{E}_c = \vec{0}$. Now, add in the cavity: no free charge is added and σ_{outer} stays the same, thus $\vec{E}_c = \vec{0}$.

<u>Corollary</u>: $\sigma_c = 0 \Rightarrow$ there is no surface charge on the cavity surfaces.
(otherwise $\Delta \vec{E}\Big|_{\text{surface of } C} \neq 0$

$\Rightarrow$ shielding E-fields is easy $\Rightarrow$ just make a <u>conducting box</u>.
(i.e. a Faraday cage)
↳ In practice, a conducting mesh is often sufficient.

<u>Case 2</u>: Consider a charge $+q$ inside the cavity.
In this case, $\vec{E}_c \neq \vec{0}$ and $\vec{E}_{outside} \neq \vec{0}$.



σ_{inner} cancels the effect of the charge q inside the conductor so that $\vec{E}_{\mathbb{V}} = \vec{0}$ (inside conductor)

↳ Thus σ_{inner} depends on the cavity surface and the position of q.

σ_{outer} is distributed so as to not produce an E-field inside the conductor on its own (i.e. independently of σ_{inner}) ⟶ σ_{outer} depends only on the outer surface of $\mathbb{V}$ (and the magnitude of q).

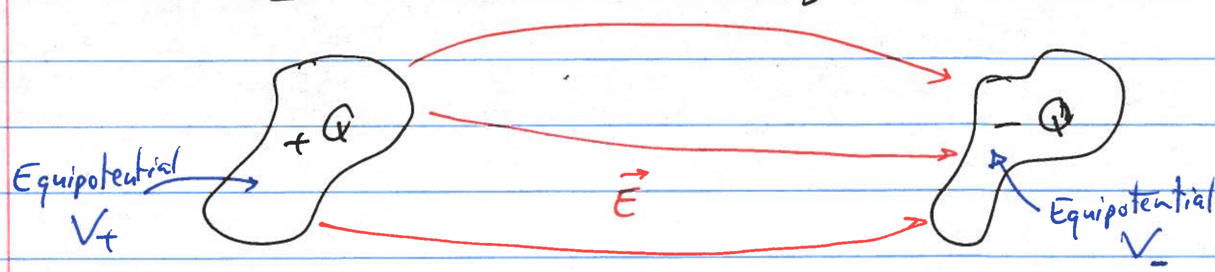
argument: remove charge $+q$ from cavity, and add total charge $+q$ to the outer surface of $\mathbb{V}$ (lots of little charges)

↳ these little charges will arrange themselves to make σ_{outer} in which $\vec{E}_{\mathbb{V}} = \vec{0}$ (solution exists)

[in this case $\vec{E}_c = \vec{0}$ and $\sigma_{inner} = 0$]

Capacitors [chpt 2.5.4]

Consider two conductors with charge $+Q$ and $-Q$.



The E-field strength is proportional to Q, the potential difference is proportional to Q $\Leftrightarrow$

$$\begin{cases} \vec{E} \propto Q \\ \Delta V = (V_+ - V_-) \propto Q \end{cases}$$