PHYS 402: Electricity & Magnetism II
Due date: Thursday, November 10, 2011

Problem set #8

Griffiths (3rd Ed.) problems
10.18, 10.22, 10.23, 11.3, 11.4, 11.6

Lagrangian and Hamiltonian of a non-relativistic charged particle

While the electromagnetic force on a charged particle cannot be derived exclusively from a scalar potential (as required for standard Lagrangian mechanics), in this problem you will show that there exists a Lagrangian functional that produces the correct equations of motion for a charged particle in an arbitrary electromagnetic field.

a) Write down the Lorentz force law for charged particle in an electric field $\vec{E}(\vec{r},t)$ and magnetic field $\vec{B}(\vec{r},t)$. Write down the Lorentz force law in terms of the electric potential $V(\vec{r},t)$ and the vector potential $\vec{A}(\vec{r},t)$.

b) Show that the Lagrangian functional **L** below produces the Lorentz force law for a particle of charge **q** and mass **m** when it is used in conjunction with the Lagrange equation of motion (i.e. the equations that follow from the principle of least action):

$$L = \frac{1}{2}m\dot{\vec{r}}^2 + q\dot{\vec{r}} \cdot \vec{A}(\vec{r},t) - qV(\vec{r},t)$$

c) Derive an expression for the canonical momentum, $\vec{p}$, and show that the Hamiltonian **H** for the charged particle can be written as

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2 + qV(\vec{r},t).$$