Dipole Radiation

Last time we derived for a small oscillating electric dipole: $d \ll \lambda \ll r$

$$\vec{E} = -\frac{p_0}{4\pi\epsilon_0}\frac{\omega^2}{c^2}\frac{\sin\theta}{r}\cos[\omega(t - r/c)]\hat{\theta}$$

$$\vec{B} = -\frac{p_0}{4\pi\epsilon_0}\frac{\omega^2}{c^3}\frac{\sin\theta}{r}\cos[\omega(t - r/c)]\hat{\phi}$$

$\vec{p}_0 = q_0\vec{d}$ = dipole moment

ω = oscillation frequency

r = dipole-observer distance

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$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}\left(\frac{p_0}{4\pi\epsilon_0}\right)^2\frac{\omega^4}{c^5}\left(\frac{\sin\theta}{r}\right)^2\cos^2[\omega(t - r/c)]\hat{r}$$

averages to 1/2

$$Intensity = \langle S\rangle = \frac{p_0^2}{32\pi^2\epsilon_0}\frac{\omega^4}{c^3}\frac{\sin^2\theta}{r^2}\hat{r}$$

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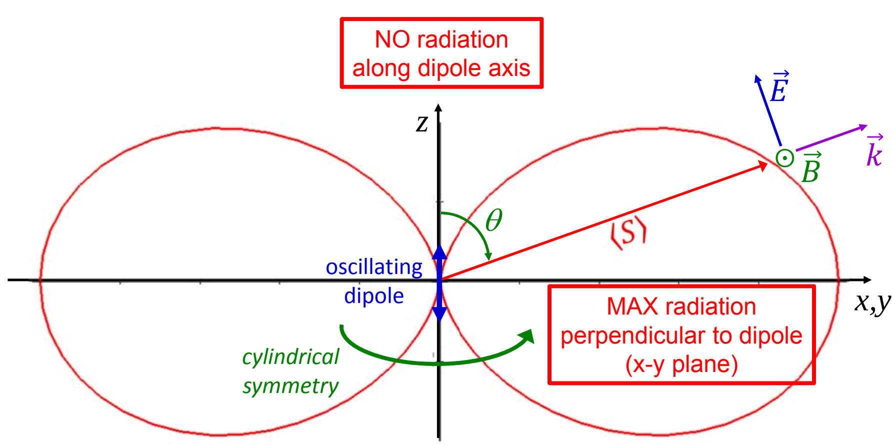
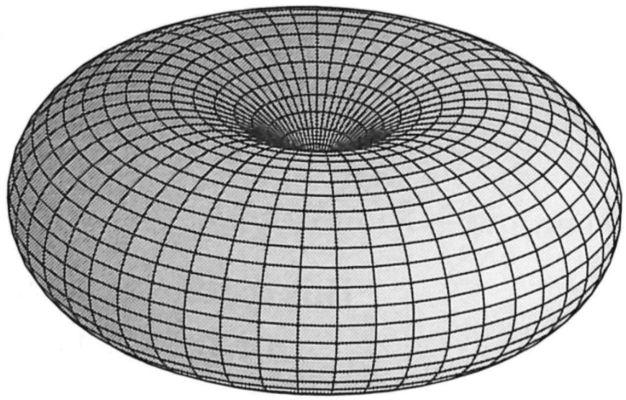
Dipole Radiation Pattern



$$Intensity = \langle S\rangle = \frac{p_0^2}{32\pi^2\epsilon_0}\frac{\omega^4}{c^3}\frac{\sin^2\theta}{r^2}\hat{r}$$

$$\propto \omega^4\frac{1}{r^2}$$

Dipole Radiation Pattern



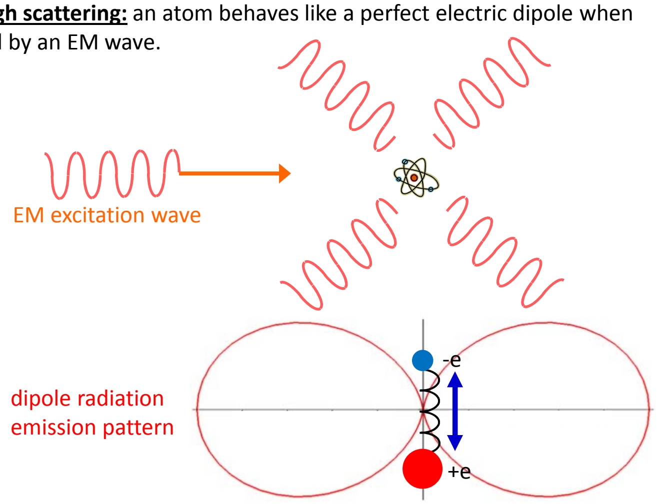
[Figure 11.4, *Introduction to Electrodynamics*, by D. Griffiths, 4th Ed.]

$$Intensity = \langle S \rangle = \frac{p_0^2}{32\pi^2\epsilon_0} \frac{\omega^4}{c^3} \frac{\sin^2\theta}{r^2} \hat{r}$$

$$\propto \omega^4 \frac{1}{r^2}$$

Dipole Radiation Example #1
Atomic fluorescence & photon scattering

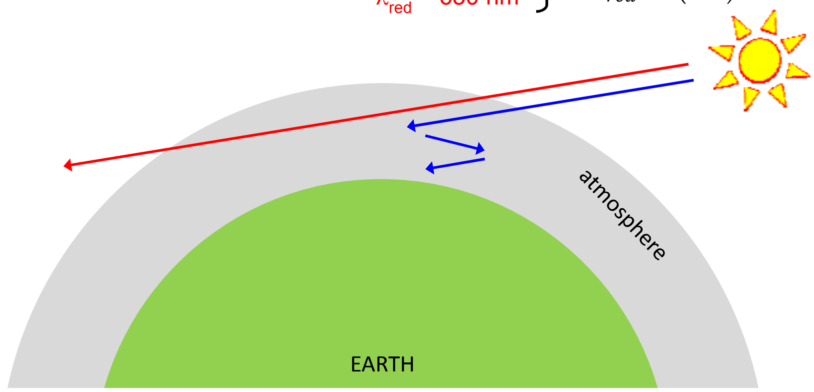
Rayleigh scattering: an atom behaves like a perfect electric dipole when excited by an EM wave.



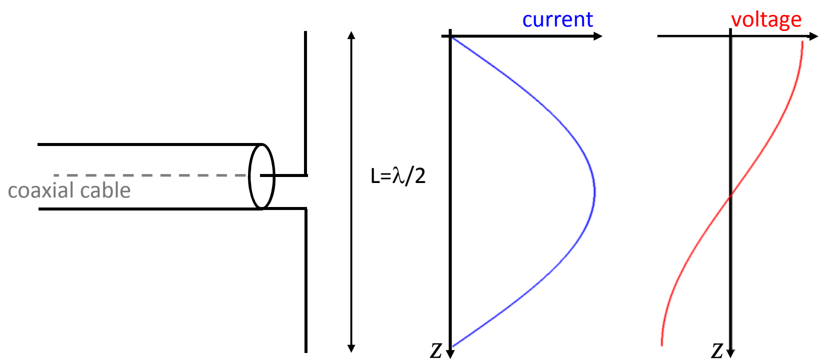
Dipole Radiation Example #2
Blue Sky

Blue light scatters at a higher rate than **red light** → Sky looks blue.

$$Intensity \propto \omega^4 \propto \frac{1}{\lambda^4} \Rightarrow \left.\begin{array}{l}\lambda_{blue} = 450\text{ nm}\\ \lambda_{red} = 650\text{ nm}\end{array}\right\} \frac{I_{blue}}{I_{red}} = \left(\frac{650}{450}\right)^4 \approx 4.3$$

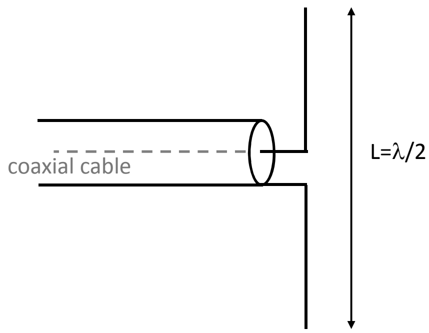


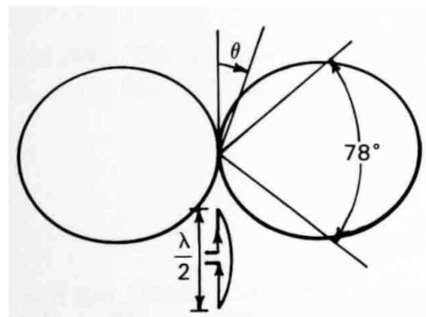
Dipole Radiation Example #3
Half-wave dipole antenna



Electric dipole antennas receive and broadcast most efficiently when the antenna size is $L=\lambda/2$ -- n.b. approximation #2 ($d \ll \lambda$) fails.

Dipole Radiation Example #3
Half-wave dipole antenna





dipole-like radiation pattern
[J.D. Krauss, *Antennas*, 2nd Ed., McGraw-Hill, 1988]

Electric dipole antennas receive and broadcast most efficiently when the antenna size is $L=\lambda/2$ -- n.b. approximation #2 ($d<<\lambda$) fails.

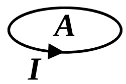
Dipole Radiation Example #3-bis
Consumer Antennas

More recently, some common antennas are **<u>not</u>** half-wave (due to space limitations), and some are not even designed to operate in the far-field.

Bluetooth @ 2.4 GHz (λ=12.5 cm)
→ designed for near-field and intermediate field operation.

Cell phones @ 800 MHz (λ=37.5 cm)
@ 1800 MHz (λ=16.7 cm)
→ These operate in the far-field, but often without a $\lambda/2$ antenna.

Magnetic Dipole Radiation



→ magnetic dipole = $m_0 = I_0 A = I_0 \pi\, radius^2$

$$\vec{E} = \frac{\mu_0 m_0}{4\pi c}\omega^2 \frac{\sin\theta}{r}\cos[\omega(t - r/c)]\hat{\phi}$$

$$\vec{B} = -\frac{\mu_0 m_0}{4\pi c^2}\omega^2 \frac{\sin\theta}{r}\cos[\omega(t - r/c)]\hat{\theta}$$

$$Intensity \propto \omega^4 \frac{1}{r^2}$$