Dipole Radiation

Last time we derived for a small oscillating electric dipole: $d \ll \lambda \ll r$

$$\vec{E} = -\frac{p_0}{4\pi\epsilon_0} \frac{\omega^2 \sin \theta}{c^2} \frac{1}{r} \cos[\omega(t - r/c)] \hat{\theta}$$

$$\vec{B} = -\frac{p_0}{4\pi\epsilon_0} \frac{\omega^2 \sin \theta}{c^3} \frac{1}{r} \cos[\omega(t - r/c)] \hat{\phi}$$

$p_0 = q_0 \hat{d}$ = dipole moment

$\omega = \text{oscillation frequency}$

$r = \text{dipole-observer distance}$

$$\vec{S} = \hat{E} \times \hat{B} = \frac{1}{\mu_0} \left( \frac{p_0}{4\pi\epsilon_0} \right)^2 \frac{\omega^4}{c^5} \left( \frac{\sin \theta}{r} \right)^2 \cos^2[\omega(t - r/c)] \hat{\vec{r}}$$

averages to 1/2

Intensity $= \langle S \rangle = \frac{p_0^2}{32\pi^2\epsilon_0} \frac{\omega^4 \sin^2 \theta}{c^3 \frac{r^2}{\hat{r}}}$
Dipole Radiation

Last time we derived for a small oscillating electric dipole: $d \ll \lambda \ll r$

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$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \left( \frac{p_0}{4\pi\epsilon_0} \right)^2 \frac{\omega^4}{c^5} \left( \frac{\sin \theta}{r} \right)^2 \cos^2[\omega(t - r/c)] \hat{r}$$

Averages to $1/2$

Intensity = $\langle S \rangle = \frac{\mu_0 p_0^2}{32\pi^2} \frac{\omega^4 \sin^2 \theta}{c^2} \frac{1}{r^2} \hat{r}$

Dipole Radiation Pattern

Intensity = $\langle S \rangle = \frac{p_0^2}{32\pi^2\epsilon_0} \frac{\omega^4 \sin^2 \theta}{c^2} \frac{1}{r^2} \hat{r}$

$\propto \omega^4 \frac{1}{r^2}$
Dipole Radiation Pattern

\[ \text{Intensity} = \langle \mathcal{S} \rangle = \frac{p_0^2}{32\pi^2\varepsilon_0 c^3} \frac{\omega^4 \sin^2 \theta}{r^2} \hat{r} \propto \omega^4 \frac{1}{r^2} \]

[Figure 11.4, Introduction to Electrodynamics, by D. Griffiths, 4th Ed.]

Dipole Radiation Example #1
Atomic fluorescence & photon scattering

**Rayleigh scattering:** an atom behaves like a perfect electric dipole when excited by an EM wave.
Dipole Radiation Example #2
Blue Sky

Blue light scatters at a higher rate than red light \( \rightarrow \) Sky looks blue.

\[
\text{Intensity} \propto \omega^4 \propto \frac{1}{\lambda^3} \quad \Rightarrow \quad \lambda_{\text{blue}} = 450 \text{ nm} \quad \lambda_{\text{red}} = 650 \text{ nm} \]

\[
\begin{align*}
I_{\text{blue}} & = \left( \frac{650}{450} \right)^4 \\
I_{\text{red}} & = 1
\end{align*}
\]

EARTH

Dipole Radiation Example #3
Half-wave dipole antenna

Electric dipole antennas receive and broadcast most efficiently when the antenna size is \( L = \frac{\lambda}{2} \) -- n.b. approximation #2 \((d<<\lambda)\) fails.
Dipole Radiation Example #3

Half-wave dipole antenna

Electric dipole antennas receive and broadcast most efficiently when the antenna size is \( L = \lambda/2 \) -- n.b. approximation #2 (\( d \ll \lambda \)) fails.

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Dipole Radiation Example #3-bis

Consumer Antennas

More recently, some common antennas are not half-wave (due to space limitations), and some are not even designed to operate in the far-field.

Bluetooth @ 2.4 GHz (\( \lambda = 12.5 \text{ cm} \))
\( \rightarrow \) designed for near-field and intermediate field operation.

Cell phones @ 800 MHz (\( \lambda = 37.5 \text{ cm} \))
@ 1800 MHz (\( \lambda = 16.7 \text{ cm} \))
\( \rightarrow \) These operate in the far-field, but often without a \( \lambda/2 \) antenna.
Magnetic Dipole Radiation

\[ m_0 \rightarrow \text{magnetic dipole} = m_0 I = I_0 \pi \text{ radius}^2 \]

\[ \vec{E} = \frac{\mu_0 m_0}{4\pi c} \omega^2 \frac{\sin \theta}{r} \cos[\omega (t - r/c)] \hat{\phi} \]

\[ \vec{B} = -\frac{\mu_0 m_0}{4\pi c^2} \omega^2 \frac{\sin \theta}{r} \cos[\omega (t - r/c)] \hat{\theta} \]

Intensity \( \propto \omega^4 \frac{1}{r^2} \)