

Dipole Radiation

Last time we derived for a small oscillating electric dipole: $d \ll \lambda \ll r$

$$\vec{E} = -\frac{p_0}{4\pi\epsilon_0} \frac{\omega^2 \sin \theta}{c^2 r} \cos[\omega(t - r/c)] \hat{\theta}$$

$$\vec{B} = -\frac{p_0}{4\pi\epsilon_0} \frac{\omega^2 \sin \theta}{c^3 r} \cos[\omega(t - r/c)] \hat{\phi}$$

$\vec{p}_0 = q_0 \vec{d}$ = dipole moment

ω = oscillation frequency

r = dipole-observer distance

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$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \left(\frac{p_0}{4\pi\epsilon_0} \right)^2 \frac{\omega^4}{c^5} \left(\frac{\sin \theta}{r} \right)^2 \cos^2[\omega(t - r/c)] \hat{r}$$

averages to 1/2

$$\text{Intensity} = \langle S \rangle = \frac{p_0^2}{32\pi^2\epsilon_0} \frac{\omega^4 \sin^2 \theta}{c^3 r^2} \hat{r}$$

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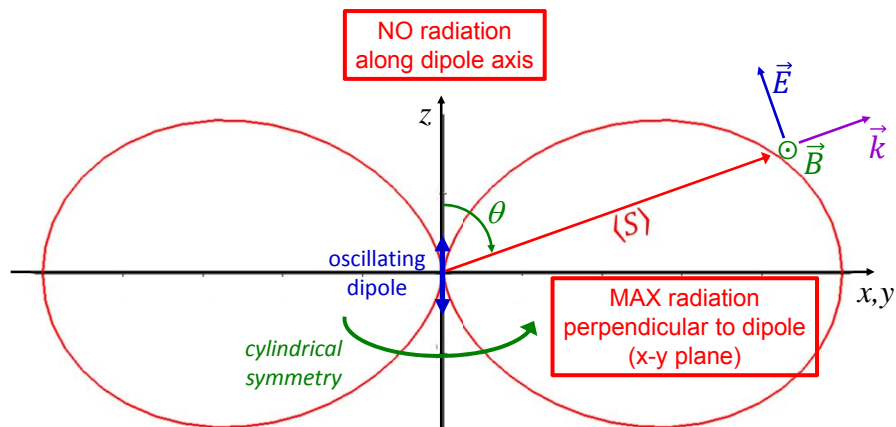
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averages to 1/2

$$\text{Intensity} = \langle S \rangle = \frac{\mu_0 p_0^2 \omega^4 \sin^2\theta}{32\pi^2 c r^2} \hat{r}$$

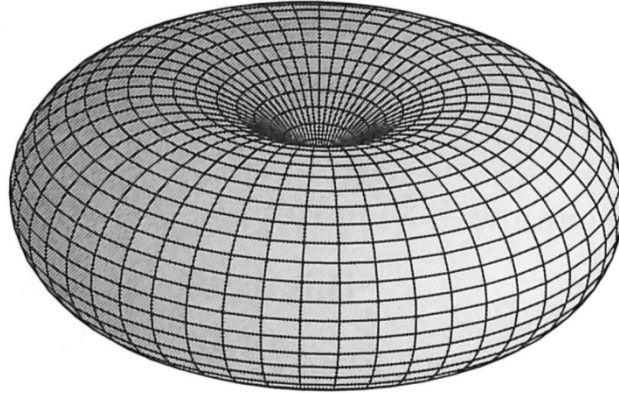
Dipole Radiation Pattern



$$\text{Intensity} = \langle S \rangle = \frac{p_0^2}{32\pi^2\epsilon_0} \frac{\omega^4 \sin^2\theta}{c^3 r^2} \hat{r}$$

$$\propto \omega^4 \frac{1}{r^2}$$

Dipole Radiation Pattern



[Figure 11.4, *Introduction to Electrodynamics*, by D. Griffiths, 4th Ed.]

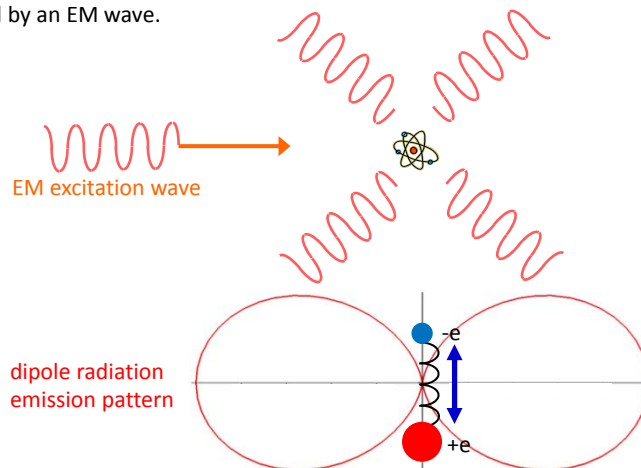
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Dipole Radiation Example #1

Atomic fluorescence & photon scattering

Rayleigh scattering: an atom behaves like a perfect electric dipole when excited by an EM wave.

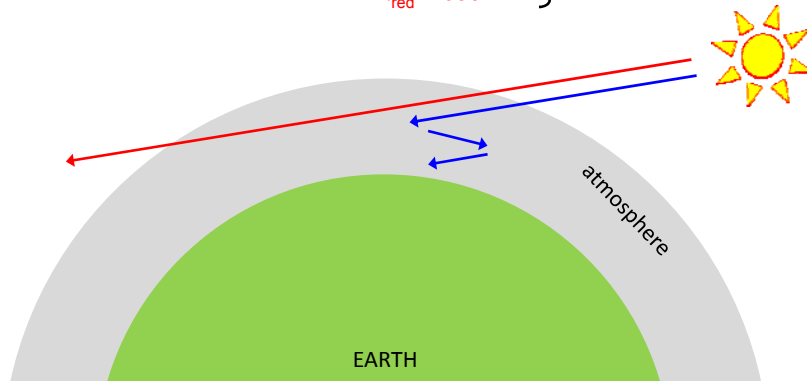


Dipole Radiation Example #2

Blue Sky

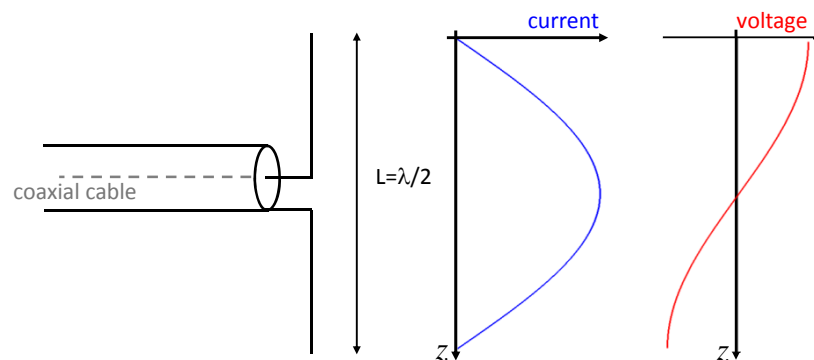
Blue light scatters at a higher rate than red light → Sky looks blue.

$$Intensity \propto \omega^4 \propto \frac{1}{\lambda^4} \Rightarrow \left. \begin{array}{l} \lambda_{blue} = 450 \text{ nm} \\ \lambda_{red} = 650 \text{ nm} \end{array} \right\} \frac{I_{blue}}{I_{red}} = \left(\frac{650}{450} \right)^4 \approx 4.3$$



Dipole Radiation Example #3

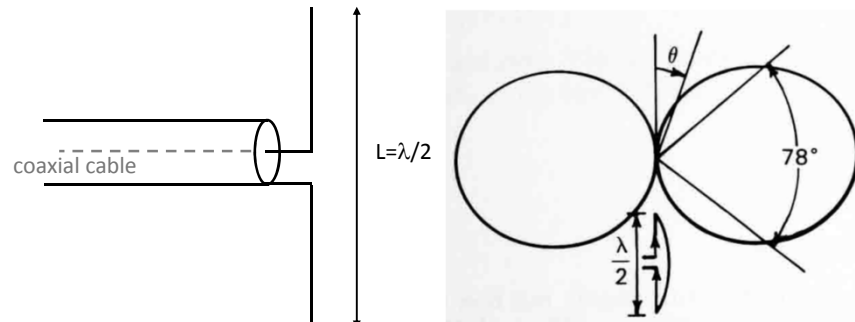
Half-wave dipole antenna



Electric dipole antennas receive and broadcast most efficiently when the antenna size is $L = \lambda/2$ -- n.b. approximation #2 ($d \ll \lambda$) fails.

Dipole Radiation Example #3

Half-wave dipole antenna



dipole-like radiation pattern

[J.D. Krauss, *Antennas*, 2nd Ed., McGraw-Hill, 1988]

Electric dipole antennas receive and broadcast most efficiently when the antenna size is $L=\lambda/2$ -- n.b. approximation #2 ($d \ll \lambda$) fails.

Dipole Radiation Example #3-bis

Consumer Antennas

More recently, some common antennas are **not** half-wave (due to space limitations), and some are not even designed to operate in the far-field.

Bluetooth @ 2.4 GHz ($\lambda=12.5$ cm)


→ designed for near-field and intermediate field operation.

Cell phones @ 800 MHz ($\lambda=37.5$ cm)

@ 1800 MHz ($\lambda=16.7$ cm)

→ These operate in the far-field, but often without a $\lambda/2$ antenna.

Magnetic Dipole Radiation


 \rightarrow magnetic dipole = $m_0 = I_0 A = I_0 \pi \text{ radius}^2$

$$\vec{E} = \frac{\mu_0 m_0}{4\pi c} \omega^2 \frac{\sin \theta}{r} \cos[\omega(t - r/c)] \hat{\phi}$$

$$\vec{B} = -\frac{\mu_0 m_0}{4\pi c^2} \omega^2 \frac{\sin \theta}{r} \cos[\omega(t - r/c)] \hat{\theta}$$

$Intensity \propto \omega^4 \frac{1}{r^2}$

