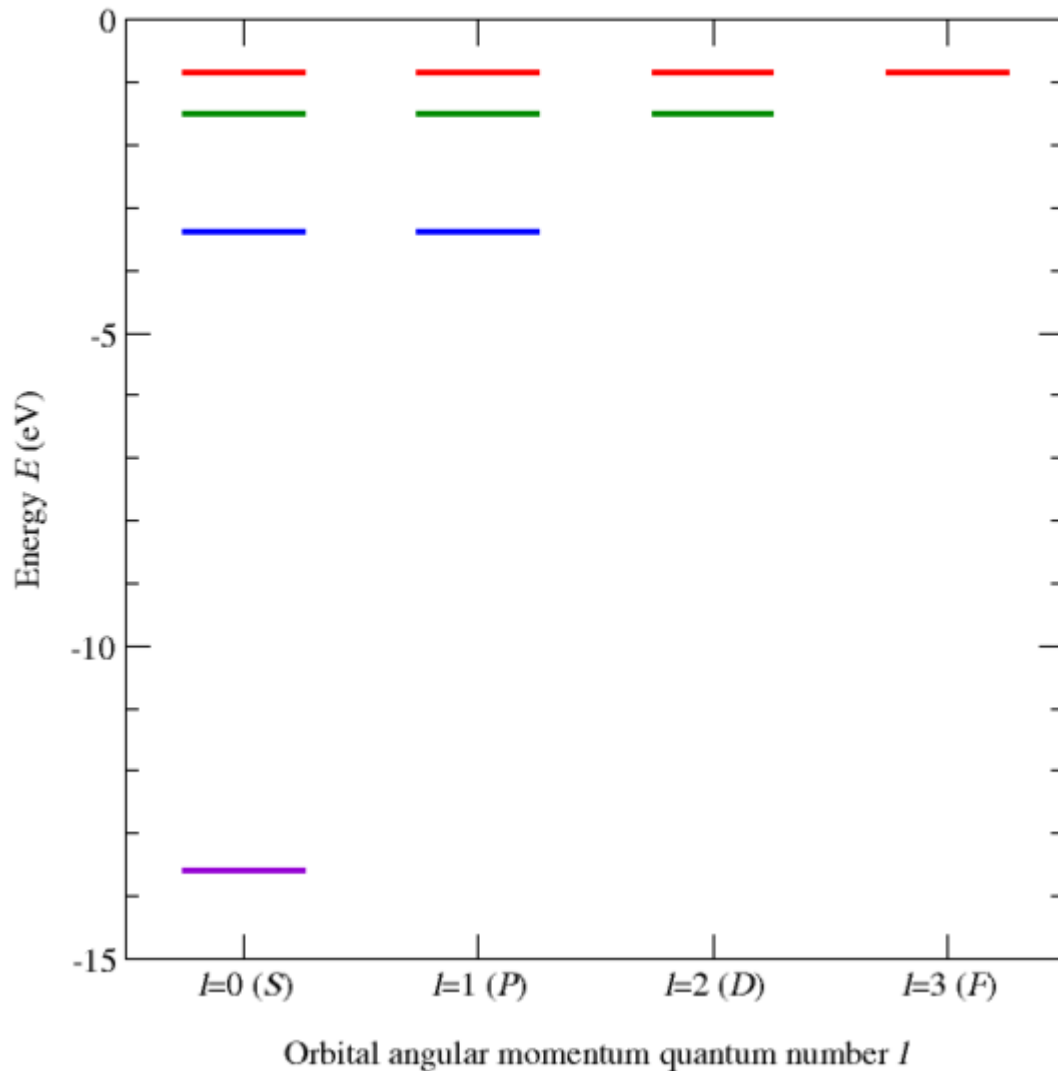


Basic Energy Levels

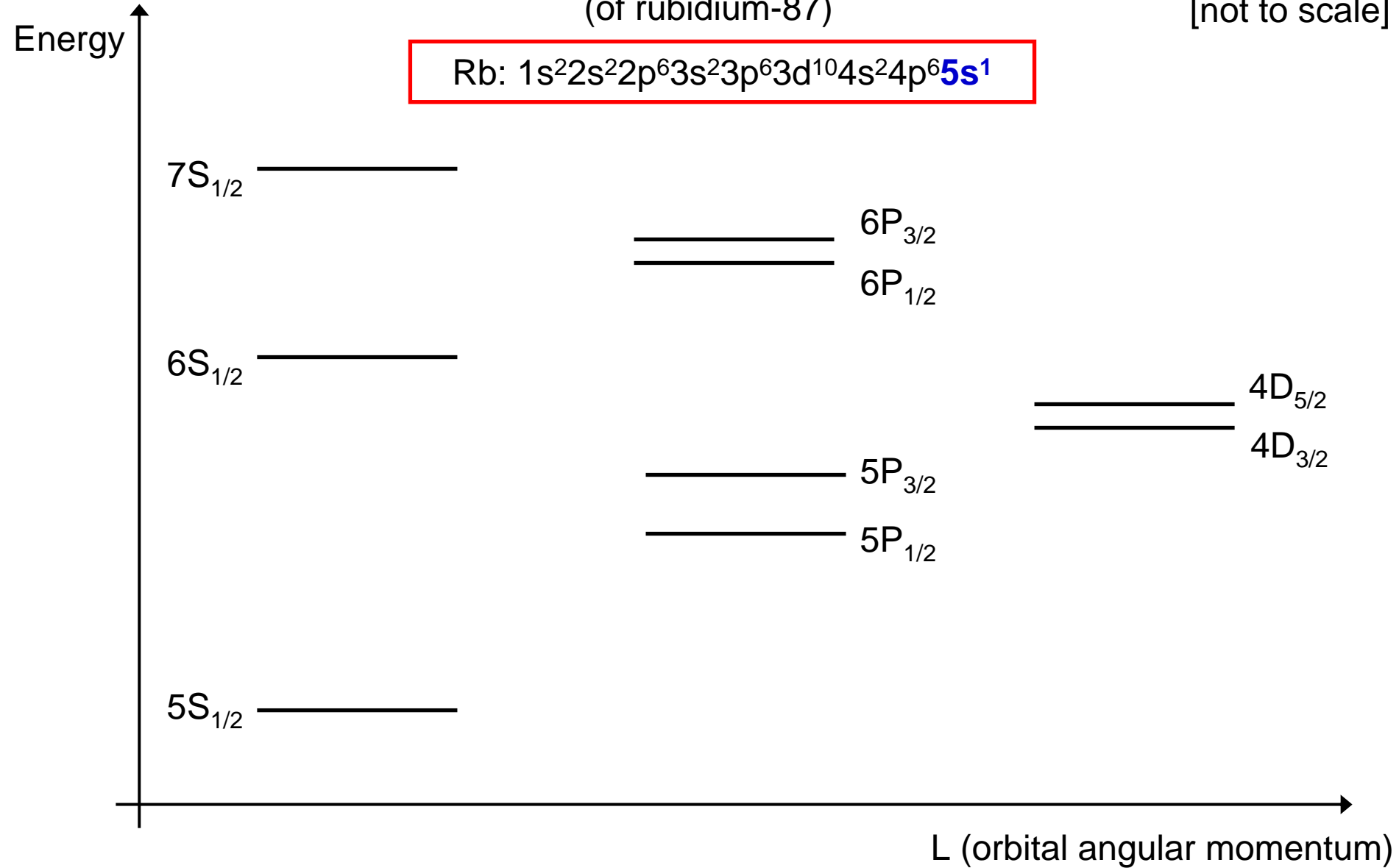
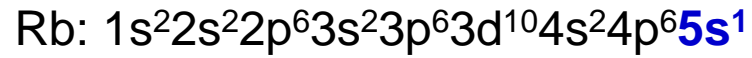
Energy Levels of Hydrogen ($n=1-4$)



Fine Structure

(of rubidium-87)

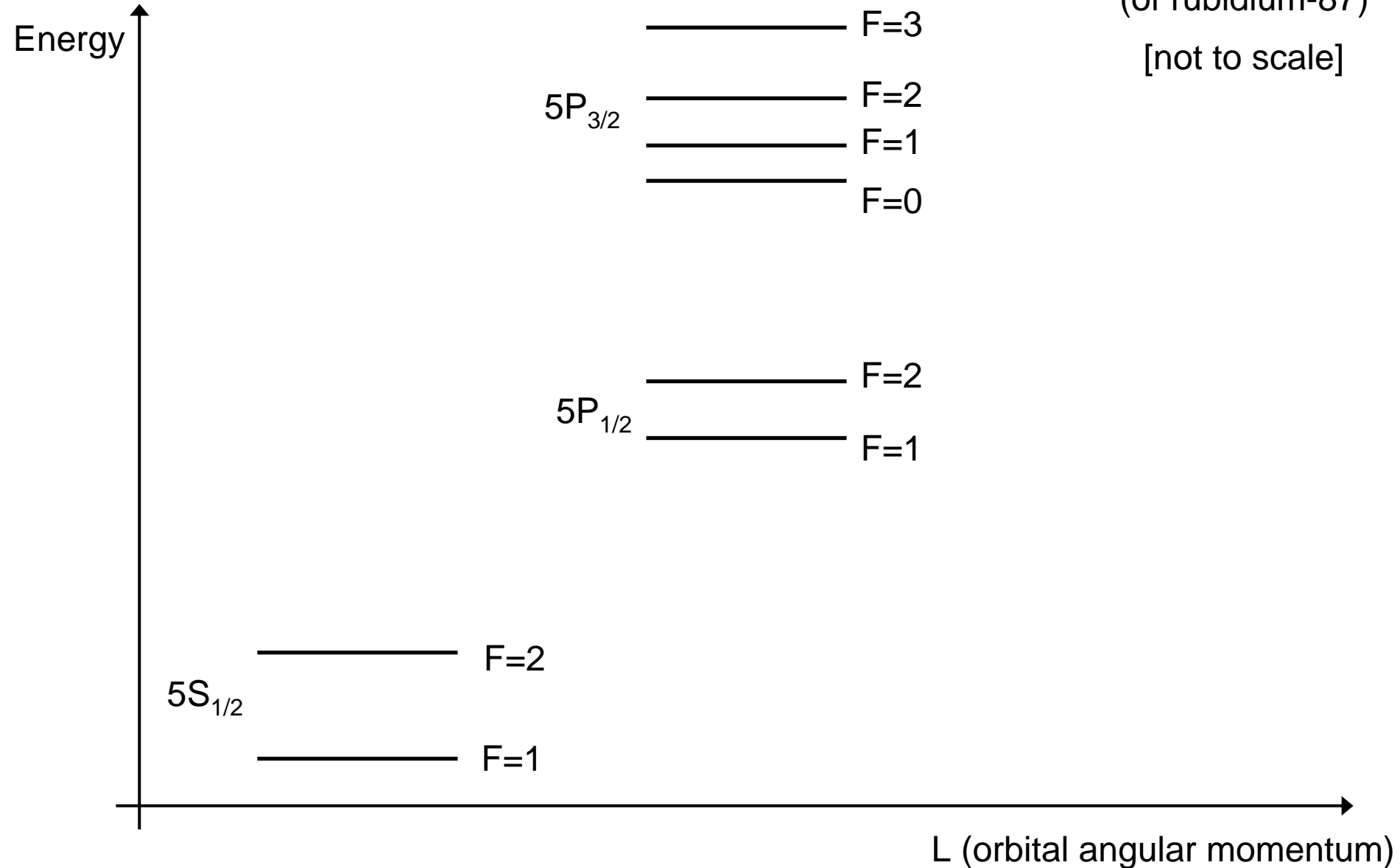
[not to scale]



Hyperfine Structure

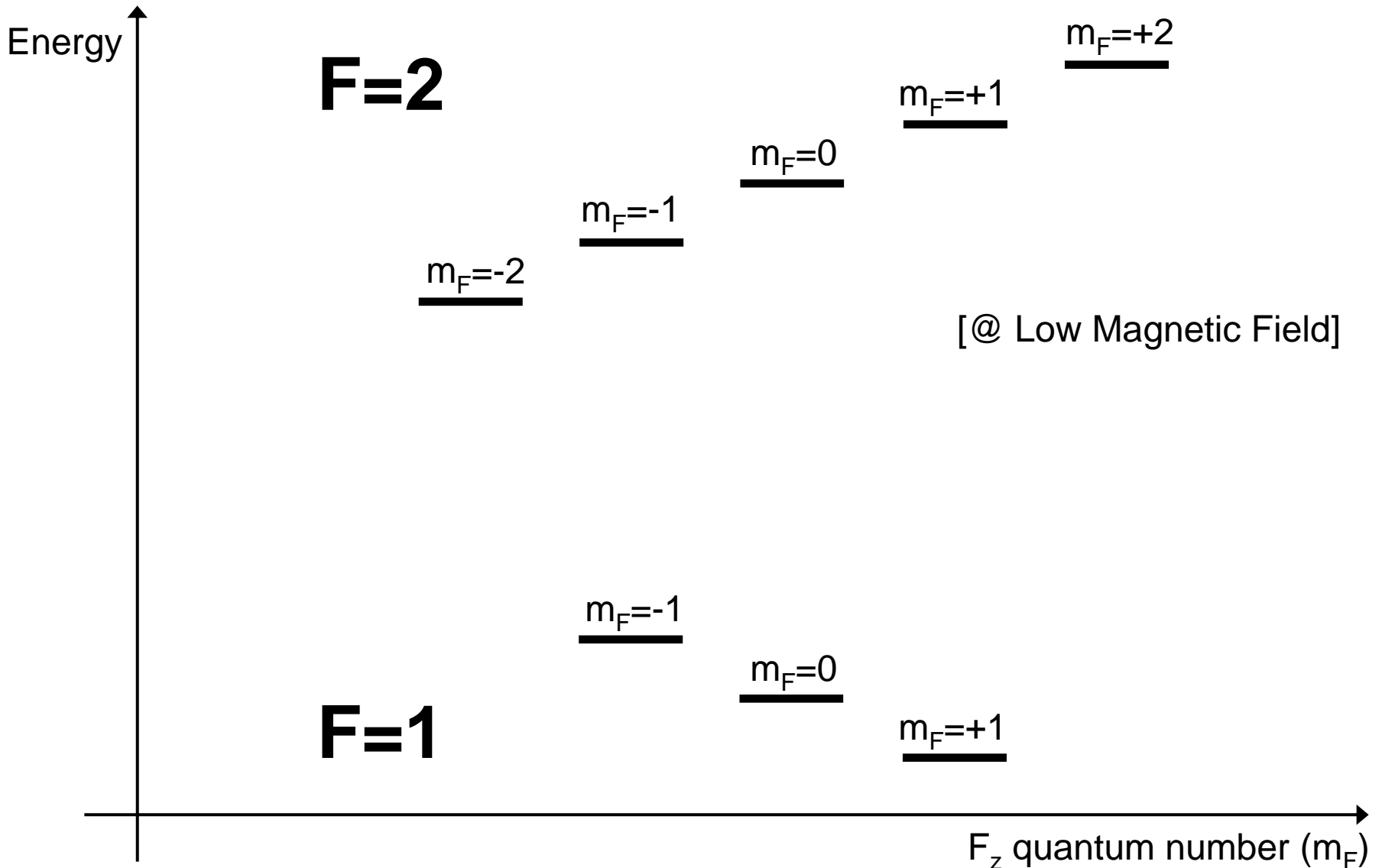
(of rubidium-87)

[not to scale]

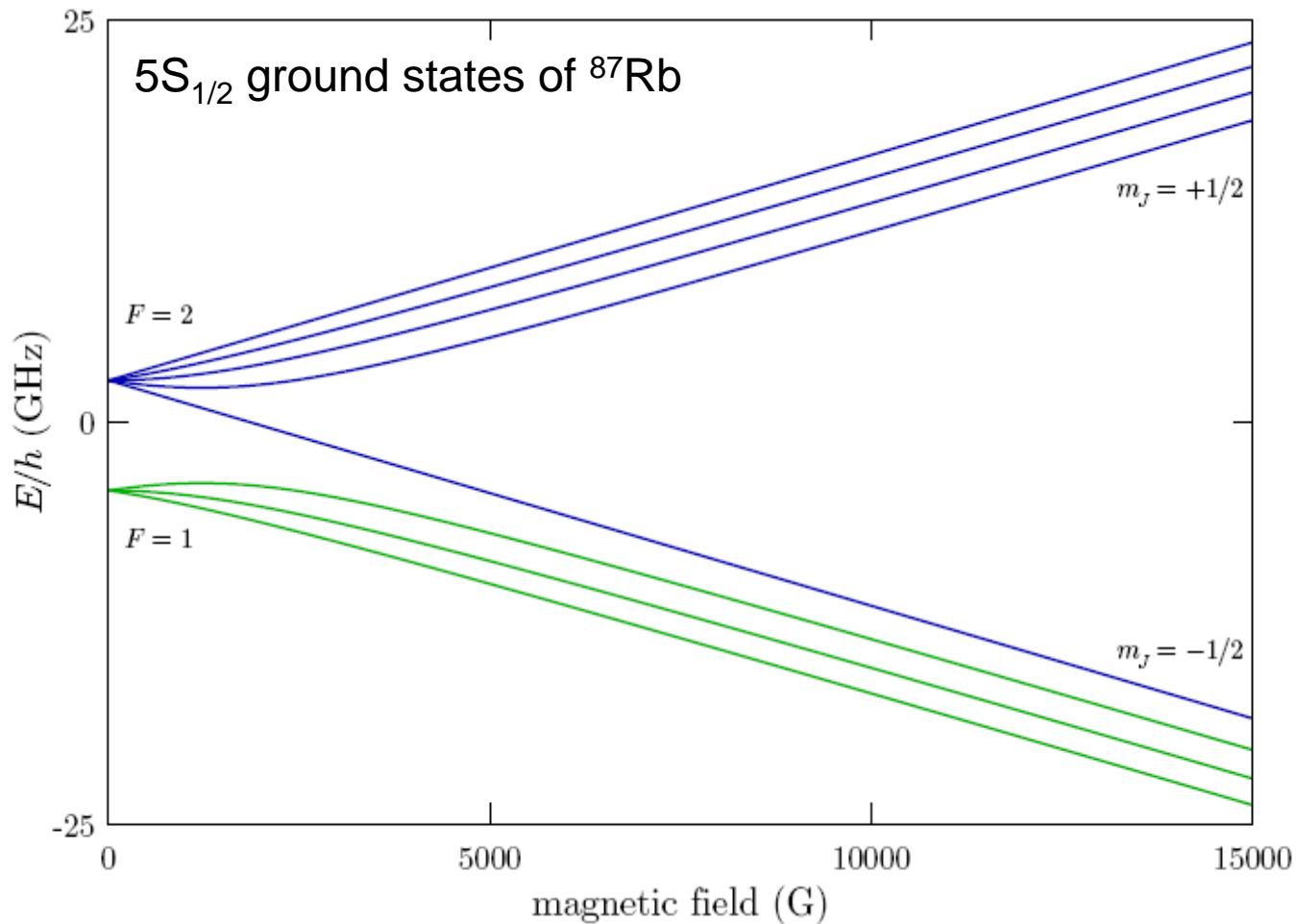


Zeeman Sub-Structure

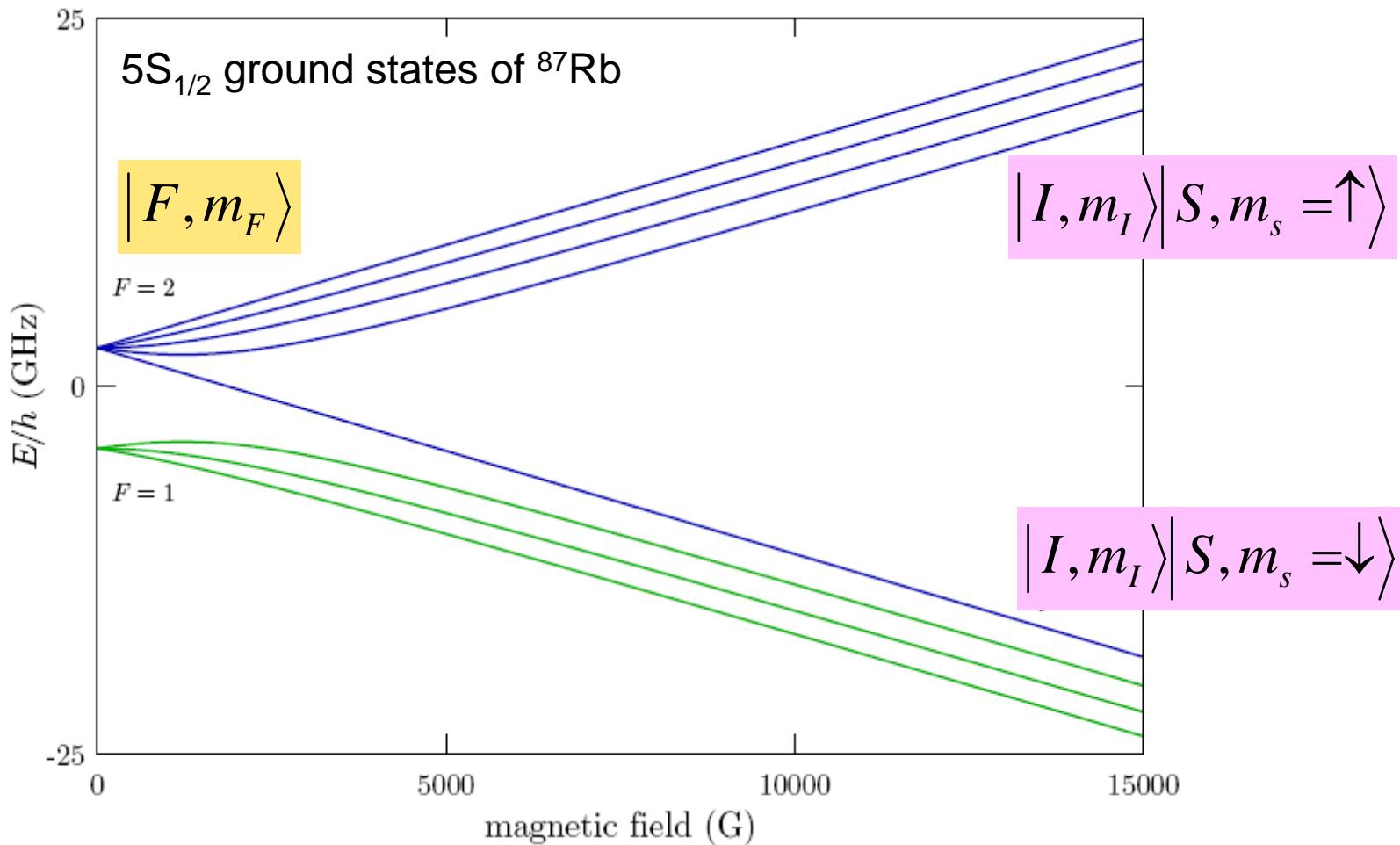
[^{87}Rb , ^{39}K , ^{41}K]



Zeeman Sub-Structure at High B-field

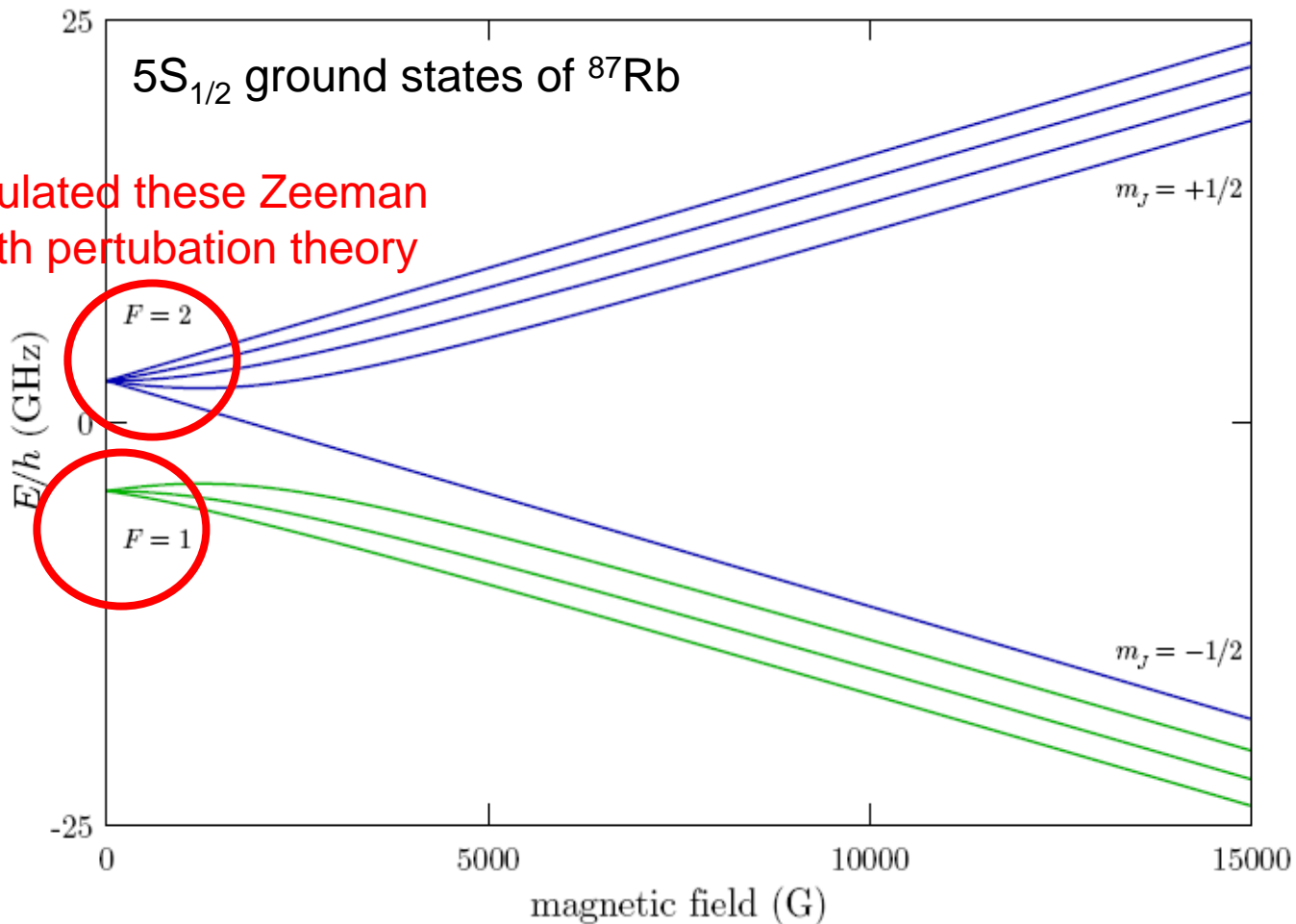


Zeeman Sub-Structure at High B-field



Zeeman Sub-Structure at High B-field

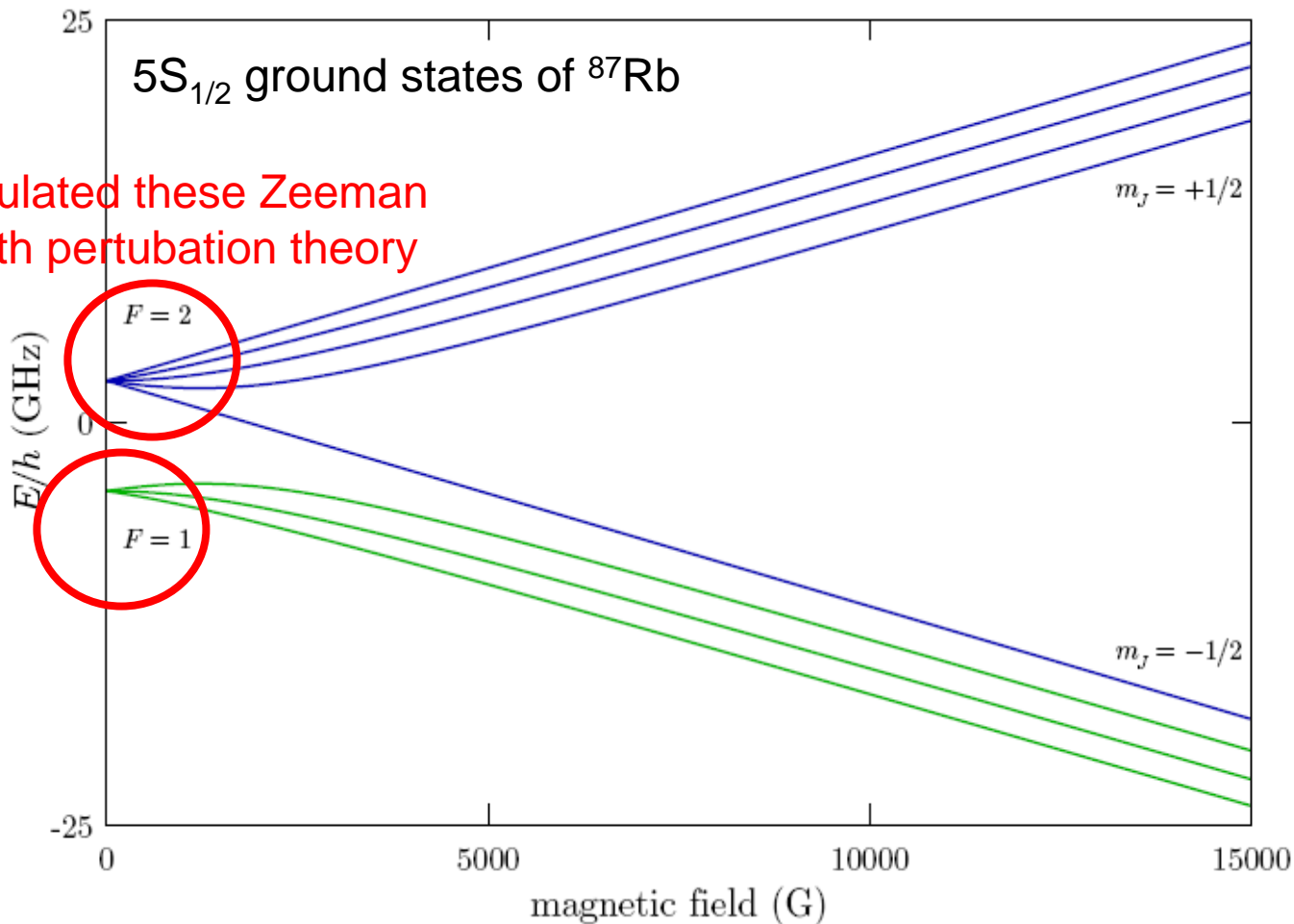
We calculated these Zeeman shifts with perturbation theory



Zeeman Sub-Structure at High B-field

How do you calculate this entire plot?

We calculated these Zeeman shifts with perturbation theory



Zeeman shifts and the Hyperfine Hamiltonian

In the ground state (S state, so $L=0$):

$$H = H_0 + H_{\text{FineStructure}} + H_{\text{Hyperfine}} + H_{\text{Zeeman}}$$

Zeeman shifts and the Hyperfine Hamiltonian

In the ground state (S state, so $L=0$):

$$H = H_0 + H_{\text{FineStructure}} + H_{\text{Hyperfine}} + H_{\text{Zeeman}}$$

$$H = \frac{P^2}{2m} - \frac{e^2}{R} + \frac{e^2}{mc^2} \frac{1}{R^3} (\vec{L} \cdot \vec{S}) + \underbrace{hA(\vec{I} \cdot \vec{J}) + \frac{\mu_B}{\hbar} (g_s \vec{S} + g_I \vec{I})}_{\text{perturbation}}$$

Breit-Rabi Formula

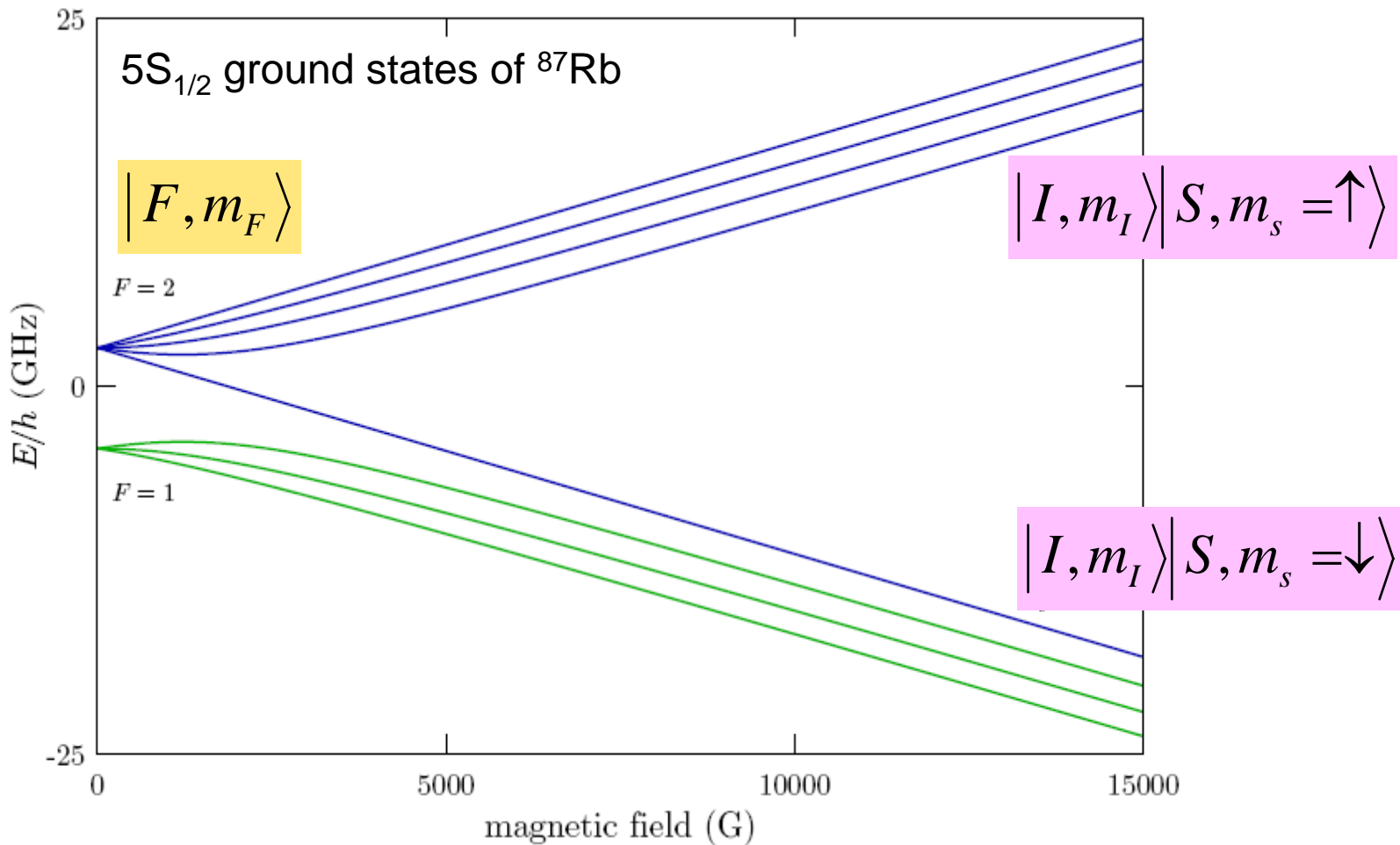
The Breit-Rabi formula for the Zeeman shift of atomic ground states is given by:

$$U(m_F, B) = g_I \mu_B m_F B + \frac{E_{hfs}}{2} \left(\pm \left(1 + \frac{4m_F x}{2I + 1} + x^2 \right)^{1/2} - \frac{1}{2I + 1} \right),$$

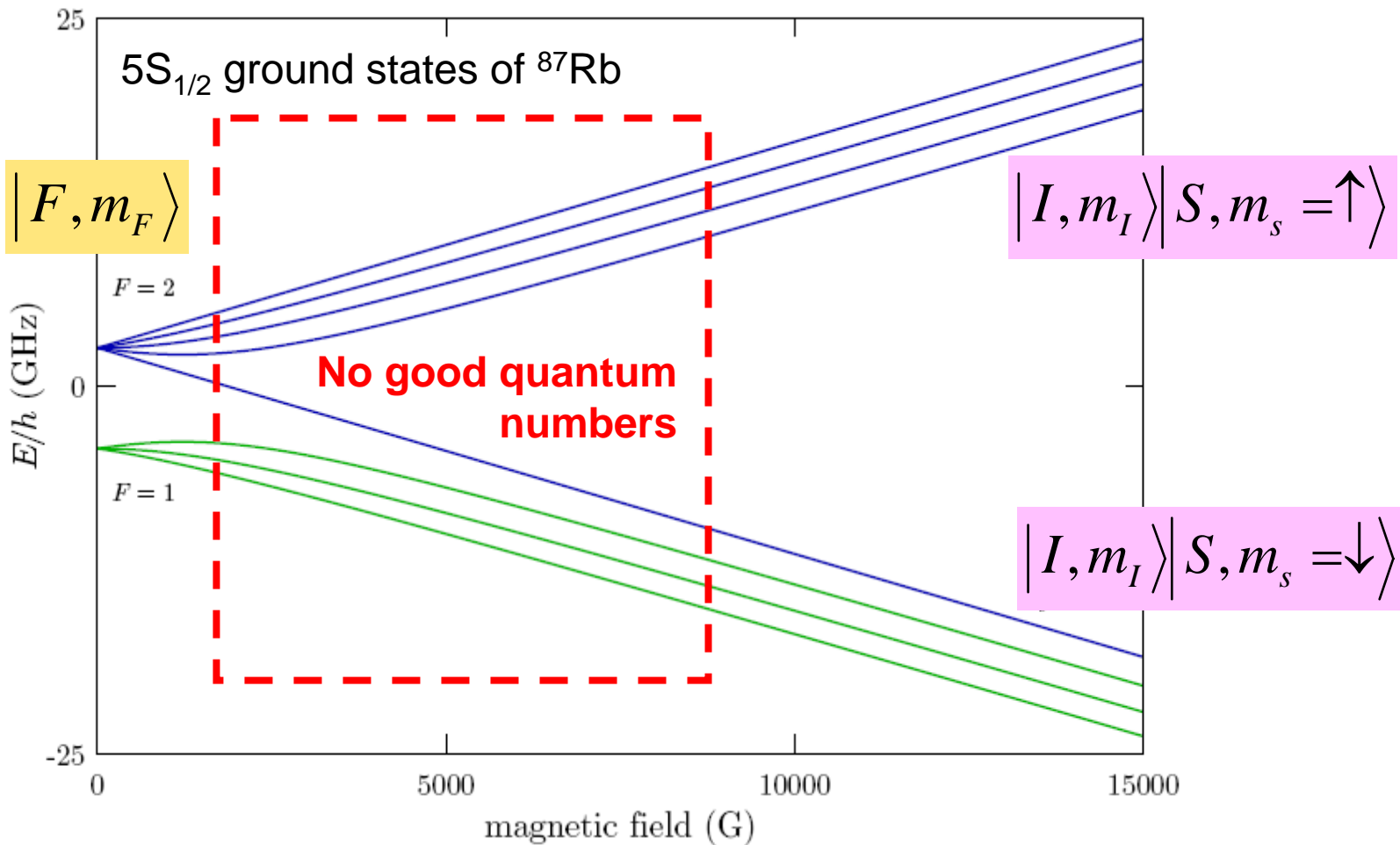
where the \pm is used for the $F = I \pm J$ state, respectively, and

$$x \equiv \frac{(g_J - g_I) \mu_B B}{E_{hfs}}.$$

Zeeman Sub-Structure at High B-field



Zeeman Sub-Structure at High B-field



[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]

Clebsch-Gordan Coefficients: $I=3/2, S=1/2$

$F=2$ with $I=3/2, S=1/2$

$$|F = 2, m_F = +2\rangle = |I = 3/2, m_I = +3/2\rangle|\uparrow\rangle$$

$$|F = 2, m_F = +1\rangle = \frac{1}{2}|m_I = +3/2\rangle|\downarrow\rangle + \frac{\sqrt{3}}{2}|m_I = +1/2\rangle|\uparrow\rangle$$

$$|F = 2, m_F = 0\rangle = \frac{1}{\sqrt{2}}|m_I = +1/2\rangle|\downarrow\rangle + \frac{1}{\sqrt{2}}|m_I = -1/2\rangle|\uparrow\rangle$$

$$|F = 2, m_F = -1\rangle = \frac{1}{2}|m_I = -3/2\rangle|\uparrow\rangle + \frac{\sqrt{3}}{2}|m_I = -1/2\rangle|\downarrow\rangle$$

$$|F = 2, m_F = -2\rangle = |m_I = -3/2\rangle|\downarrow\rangle$$

Clebsch-Gordan Coefficients: $I=3/2, S=1/2$

$F=1$ with $I=3/2, S=1/2$

$$|F=1, m_F=+1\rangle = \frac{\sqrt{3}}{2} |m_I=+3/2\rangle |\downarrow\rangle - \frac{1}{2} |m_I=+1/2\rangle |\uparrow\rangle$$

$$|F=1, m_F=0\rangle = \frac{1}{\sqrt{2}} |m_I=+1/2\rangle |\downarrow\rangle - \frac{1}{\sqrt{2}} |m_I=-1/2\rangle |\uparrow\rangle$$

$$|F=2, m_F=-1\rangle = -\frac{\sqrt{3}}{2} |m_I=-3/2\rangle |\uparrow\rangle + \frac{1}{2} |m_I=-1/2\rangle |\downarrow\rangle$$

General Formula...

$$\begin{aligned} |F_+ = I + S, m_F\rangle = \\ \frac{\sqrt{F_+ + m_F}}{\sqrt{2I + 1}} |m_I = m_F - 1/2\rangle |\uparrow\rangle + \frac{\sqrt{F_+ - m_F}}{\sqrt{2I + 1}} |m_I = m_F + 1/2\rangle |\downarrow\rangle \end{aligned}$$

$$\begin{aligned} |F_- = I - S, m_F\rangle = \\ -\frac{\sqrt{F_+ - m_F}}{\sqrt{2I + 1}} |m_I = m_F - 1/2\rangle |\uparrow\rangle + \frac{\sqrt{F_+ + m_F}}{\sqrt{2I + 1}} |m_I = m_F + 1/2\rangle |\downarrow\rangle \end{aligned}$$