Physics 404 and 690-03: Intro to AMO Physics
Due date: Friday, February 17, 2012.

Problem Set #2

1. *Wiener-Khintchine Theorem*
In class, we derived a formula for the Normalized Power Spectral Density, $F(\omega)$, in terms of the first order correlation function, $g^{(1)}(\tau)$. Derive an expression for $g^{(1)}(\tau)$ in terms of $F(\omega)$.

2. *Level crossings*
Consider a 2-level atom with ground and excited states $|g\rangle$ and $|e\rangle$, respectively, and energies, E_g and E_e, respectively. These energies depend on a parameter m such that

$$E_g(m) = E_g(0) + \alpha \cdot m \text{ and } E_e(m) = E_e(0) - \alpha \cdot m$$

We modify the basic Hamiltonian of the system, H_0, by adding a generic interaction with Hamiltonian:

$$H_{int} = \begin{bmatrix} 0 & W \\ W^* & 0 \end{bmatrix}$$

a) Calculate the new eigenenergies of the system with the interaction present, as a function of m.

b) Plot the energy of the system as function of m, with and without the interaction present.

c) Calculate the new eigenstates of the system with the interaction present, and show that one can go continuously from modified state $|g\rangle$ to modified state $|e\rangle$, and vice versa, by adiabatically varying m. What is the condition for adiabaticity? Make a quantitative (and logical) argument. What happens if you ramp m much faster than the adiabatic condition (support your answer with a quantitative argument)?

Extra graduate student problem

3. *Gaussian lineshape*

Consider a gas of identical atoms that emit light at a resonant frequency of ω_0 (when at rest). The atoms in the gas have a spread of velocities due to their finite temperature, T, which leads to a spread in the resonant frequency of the atoms due to the Doppler effect. The probability for an atom to emit light at a frequency ω close to ω_0 in a given direction is then given by

$$P(\omega) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(\omega-\omega_0)^2}{\sigma^2}\right) \quad \text{with} \quad \sigma = \omega_0\sqrt{\frac{kT}{mc^2}},$$

where m is the mass of the atom, c is the speed of light, and k is Boltzmann's constant.

We will assume that the atom is excited by some mechanism, and we look at the radiated light in a given direction. The emitted electric field in this direction is then given by

$$E(t) = E_0 \sum_{i=1}^{N} \exp(-i\omega_i t + i\phi_i)$$

where the sum is over the N atoms in the gas and the ϕ_i are random stationary phases.

a. Show that $\left\langle E^*(t)E(t+\tau)\right\rangle = E_0^2 \sum_{i=1}^{N} \exp(-i\omega_i\tau)$.

b. Show that $g^{(1)}(\tau) = \exp\left(-i\omega_0\tau - \frac{1}{2}\sigma^2\tau^2\right)$.