Week 0: Thursday, August 28, 2025

Classical E-M quantities:

$$\text{Energy} = \frac{1}{2}\int_{\substack{\text{all}\\ \text{space}}} \left[\varepsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right] d^3r \overset{?}{\doteq} H$$

$$\text{Momentum} = \vec{P} = \varepsilon_0 \int_{\substack{\text{all}\\ \text{space}}} \vec{E}\times\vec{B}\, d^3r$$

$\underbrace{\vec{E}\ \&\ \vec{B}}_{\text{Operators}}$: Wavefunctions or Operators ?

- obey the superposition principle
- oscillate, wave-like, spread out over space
- Measurable quantities $\rightarrow$ operators

From relativistic mechanics, we know:

For photons: $E^2 = p^2c^2 + \cancel{m_0^2}\, c^4$ $\quad = 0$ for photon

E = energy

$$\Rightarrow E = pc \qquad \left[\text{in E\&M}: \underbrace{\mathcal{E}}_{\text{energy density}} = \underbrace{\not{p}}_{\text{momentum density}} c\right]$$

In QM, particles have momentum $\boxed{\vec{p} = \hbar\vec{k}}$ $\quad k = \text{wavevector} = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ $\quad \lambda f = c$

$$\Rightarrow p = \hbar\frac{2\pi f}{c} = \hbar\frac{\omega}{c}$$

$$\Rightarrow E_\gamma = pc = \hbar\frac{\omega}{c}\cdot c \Rightarrow \boxed{E = \hbar\omega} \overset{?}{\doteq} H_{\text{photon}}$$

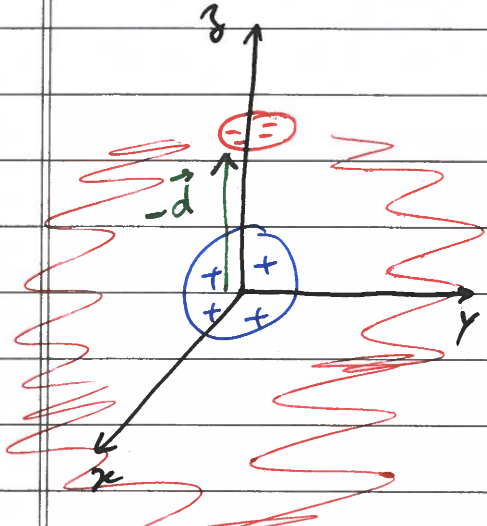
Semi-Classical Atom-light interaction

(I) Motivation

- Classical physics often contains the essential physics and then there are "quantum corrections".
- Later, when we do the quantum treatment, we will be able to distinguish more easily between classical & quantum properties of atom light interactions.
- Easier than full QM treatment.

(II) Model Atom

We consider a "semi-classical atom" with the



- { nucleus: "+" charge $= +e$
  $1e^-$: "−" charge $= -e$
- no preferred orientation of dipole
  $\vec{d} = -e\vec{r}$

So we will use an isotropic e^- cloud (nucleus is very heavy)

$\hookrightarrow \langle \vec{d} \rangle = 0 = -e\langle \vec{r} \rangle$

- Mysteriously, e^- does not spiral into the nucleus, but remains at an equilibrium distance:

$$\sqrt{\langle \vec{r}^{\,2} \rangle} = r_{equilibrium} \; (\text{but } \langle \vec{r} \rangle = 0)$$

- If the e^- is moved away from its equilibrium position, then there is a linear restoring force:

(at origin on average)

$$\vec{F} = m \frac{d^2 \vec{r}}{dt^2} = -k \vec{r}$$

$$\Leftrightarrow \ddot{\vec{r}} + \frac{k}{m} \vec{r} = 0 \Leftrightarrow \ddot{\vec{r}} + \omega^2 \vec{r} = 0$$

with $\omega = \sqrt{\frac{k}{m}}$

$\left(\text{also } \ddot{\vec{d}} + \frac{k}{m} \vec{d} = 0\right)$

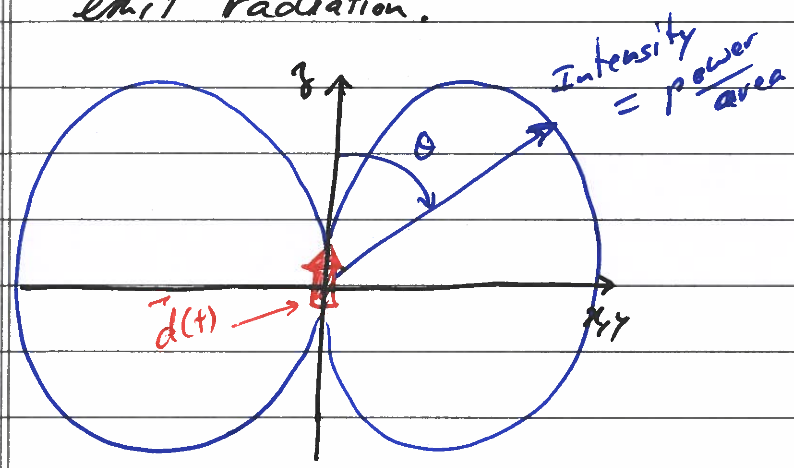
$\Rightarrow$ "simple harmonic atom"

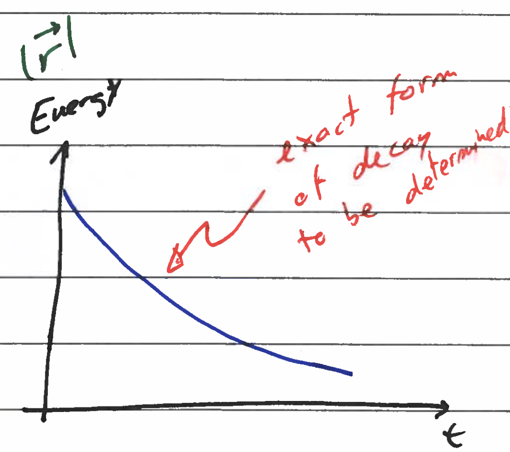
<u>Solution:</u> $\vec{r}(t) = \vec{r}_0 \sin(\omega t) + \vec{r}_0^{\,\prime} \cos(\omega t)$

with $\vec{r}_0$ and $\vec{r}_0^{\,\prime}$ defined by the initial conditions.

III Radiation Damping

Accelerating charges and oscillating dipoles (i.e., antennas) emit radiation.





$$\boxed{\text{Intensity} = \langle \text{Poynting vector} \rangle = \langle \vec{S} \rangle = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{d_0^2}{32\pi\varepsilon_0}\frac{\omega^4}{c^3}\frac{\sin^2\theta}{r^2}\hat{r}}$$

The total radiated power for an oscillating dipole $\vec{d} = \vec{d}_0 \sin(\omega t)$ is

$$\frac{d}{dt}\langle \text{Energy} \rangle = \langle \text{power} \rangle = -\frac{1}{4\pi\varepsilon_0}\frac{d_0^2\omega^4}{3c^3} = \frac{-1}{4\pi\varepsilon_0}\frac{e^2\omega^4 r_0^2}{3c^3}$$

(see Griffiths p401-405)

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What's the form of the damping?

Assume that the damping rate is much smaller than the oscillation rate.

In 1D: Energy of "harmonic" atom $= \frac{1}{2}k r_0^2 = \frac{1}{2}m\omega^2 r_0^2 = E_{Ho}$

amplitude of harmonic atom oscillations

$$\text{Energy loss rate} = \frac{d}{dt}E_{Ho} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2\omega^4 r_0^2}{3c^3}$$

$$= -2\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2\omega^2}{3mc^3}}_{\gamma}E_{Ho}$$

$$\Leftrightarrow \frac{d}{dt}E_{Ho} = -2\gamma E_{Ho} \Rightarrow \boxed{E_{Ho}(t) = E_{Ho}\Big|_{t=0} e^{-2\gamma t}}$$

If $\gamma \ll \omega$, then $r_0(t)$ varies slowly compared to oscillations:

$$\Rightarrow \quad r_0(t) = \sqrt{\frac{2\, E_{H_0,\, t=0}}{m\omega^2}}\; e^{-\gamma t}$$

$$\Rightarrow \text{ electron oscillates as } r(t) = r_0(t)\sin(\omega t) = \underbrace{A e^{-\gamma t} \sin(\omega t)}$$

damping & oscillation
are decoupled/independent