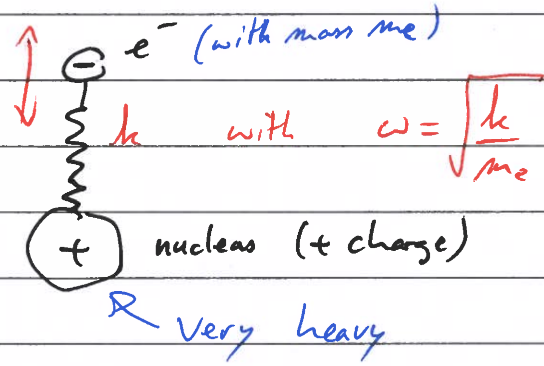
Tuesday, September 2, 2025

Semi-Classical atom-light interaction (continued)

model atom: simple harmonic atom

$\vec{d} = -e\vec{r}$

electric dipole



Dipole radiation:

The total radiated power for an oscillating dipole $\vec{d} = \vec{d}_0 \sin(\omega t)$ is

$$\frac{d}{dt}\langle \text{energy} \rangle = \langle \text{power} \rangle = -\frac{1}{4\pi\varepsilon_0}\frac{d_0^2\,\omega^4}{3c^3} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2\omega^4 r_0^2}{3c^3}$$

$\langle\ \rangle$ = average over many optical cycles

r_0 = amplitude of oscillations

[see Griffiths EM p401-405]

What's the form of the damping?

We assume that the damping rate is much smaller than the oscillation rate.

In 1D, Energy of "harmonic" atom $= \frac{1}{2} k r_0^2 = \frac{1}{2} m\omega^2 r_0^2 = E_{HO}$

amplitude of harmonic atom oscillations

Energy loss rate $= \frac{d}{dt} E_{HO} = -\frac{1}{4\pi\varepsilon_0} \frac{e^2 \omega^4 r_0^2}{3c^3}$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{2 e^2 \omega^2}{3 c^3 m} \underbrace{\frac{1}{2} m \omega^2 r_0^2}_{E_{HO}}$$

$$= -2 \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{e^2 \omega^2}{3 m c^3}}_{\gamma = \frac{1}{4\pi\varepsilon_0} \frac{e^2 \omega^2}{3 m c^3}} E_{HO}$$

$$\Longleftrightarrow \frac{d}{dt} E_{HO} = -2\gamma E_{HO} \quad \Longrightarrow \quad \boxed{E_{HO}(t) = E_{HO}\Big|_{t=0} e^{-2\gamma t}}$$

If $\gamma \ll \omega$, then $r_0(t)$ varies slowly compared to the oscillations:

$$\Longrightarrow \; r_0(t) = \sqrt{\frac{2 E_{HO,\, t=0}}{m \omega^2}} \; e^{-\gamma t}$$

$\Longrightarrow$ electron oscillates as $r(t) = r_0(t) \sin(\omega t)$

$$= \underbrace{A e^{-\gamma t} \sin(\omega t)}_{\text{damping \& oscillations are decoupled/independent}}$$

The radiation damping result suggests that the model for our semi-classical atom should include radiation damping as "viscosity" or "friction" force:

$$\boxed{\vec{F} = m\ddot{\vec{r}} = -k\vec{r} - 2m\gamma\dot{\vec{r}}}$$

not the only way to include radiation damping in our model
↳ EM suggest "$\dddot{r}$" term

where $\boxed{\gamma_{classical} = \frac{1}{4\pi\varepsilon_0}\frac{e^2\omega_0^2}{3mc^3}}$

<u>note</u>: QM gives $\gamma_{QM} \propto \omega_0^3$

↳ If you include QM properties of atom (i.e. dipole matrix element) into this classical model, then you get $\gamma \propto \omega^3$

<u>Example</u>: Rubidium atom $\lambda = 780$ nm

$\Rightarrow \omega = 2\pi \times 3.8 \times 10^{14}$ rads/s

↳ $2\gamma_{classical} = 2\pi \times 5.8$ MHz

Compare with $2\gamma_{exp} = 2\pi \times 6.0$ MHz

<u>note</u>: We assumed that $\omega \gg \gamma$ is valid. Also, we ignored that $\omega \to \sqrt{\omega^2 - \gamma^2}$.

Fortuitous agreement! In $\underline{H}$, it's a factor of 2.4 too big.

Summary of semi-classical atom

Pro:

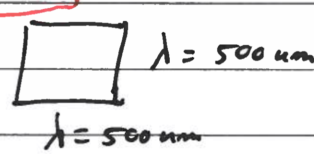
see homework #1

- Simple
- Correct lineshape: Lorentzian
- Radiative damping factor, i.e. linewidth = FWHM = 2γ is roughly correct (to within a factor ~2)
- classical, on-resonance, total scattering cross-section is exactly correct:

$$\sigma_T = \frac{\text{total scattered Power (on resonance)}}{\underbrace{\text{Intensity}}}$$

$\hookrightarrow I = \langle S \rangle = \left\langle \frac{1}{\mu_0}(\vec{E}\times\vec{B}) \right\rangle$
$= \frac{1}{2} c \varepsilon_0 E_0^2$

$$\Rightarrow \boxed{\sigma_{\substack{\text{classical}\\ \text{on-resonance}}} = \frac{3}{2\pi}\lambda^2}$$

$\simeq \frac{1}{2}$ 

note: atom diameter $\simeq 1\,\text{Å}$

- Scattered light has the same optical frequency as the incident light $\longrightarrow$ true at low intensities

Con:

- Scattering rate can become arbitrarily high ... even at high intensities $\langle P \rangle_{\text{scattered}} \propto \underset{\text{intensity}}{E_0^2}$
  $\hookrightarrow$ Experiments observe a saturation of $P_{\text{scattered}}$

not true at high intensity $\longrightarrow$

- Scattered light always has the optical frequency as incident light.
- Linewidth is independent of incident intensity
  $\hookrightarrow$ Experimentally, the linewidth becomes broader at high intensity.

Rayleigh Scattering

Let's consider the case of very far off-resonance scattering : $\omega << \omega_0$ (i.e. detuning $\delta \sim \omega_0$)

In this case :

$$\langle P_{scattered} \rangle = \frac{1}{4\pi\varepsilon_0} \frac{e^4}{3m_e^2 c^3} E_0^2 \frac{\omega^4}{(\omega_0^2 - \underbrace{\omega^2}_{small})^2 + \underbrace{4\omega^2\gamma^2}_{small}}$$

$$\simeq \frac{1}{4\pi\varepsilon_0} \frac{e^4}{3m_e^2 c^3} E_0^2 \frac{\omega^4}{\omega_0^4}$$

ω^4 scaling or $\frac{1}{\lambda^4}$ scaling

ω_0 is typically in the UV for air molecules

$\Rightarrow$ Higher frequencies scatter much more than lower frequencies

red : $\lambda_R = 650$ nm
blue : $\lambda_B = 475$ nm

$$\frac{P_{scattered,\,blue}}{P_{scattered,\,red}} = \left(\frac{\omega_B}{\omega_R}\right)^4 = \left(\frac{\lambda_R}{\lambda_B}\right)^4 = (1.37)^4 = 3.5$$

explains why sky is blue!!

multiple scatters in sky
↳ effect is compounded

I) What's Coherence?

Two or more things are said to be coherent if there exists a definite phase relationship between them, so that some form of interference effect can be observed.

ex: 1_ Optics : light in two arms of an interferometer.

2_ Quantum mechanics : any superposition of two states

$$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle$$
$$= |\alpha|\,|a\rangle + e^{i\phi}|\beta|\,|b\rangle$$

Multiple particles

$$|\psi\rangle = \left(|a\rangle_1 + e^{i\phi_1}|b\rangle_1\right)\left(|a\rangle_2 + e^{i\phi_2}|b\rangle_2\right)\cdots\left(|a\rangle_n + e^{i\phi_n}|b\rangle_n\right)$$

Coherent: $\phi_1 = \phi_2 = \cdots = \phi_n$

incoherent: $\phi_1 \neq \phi_2 \neq \cdots \neq \phi_n$

stopped here

II) Spatial coherence

Different points of an optical wavefront share a definite, fixed phase relationship.