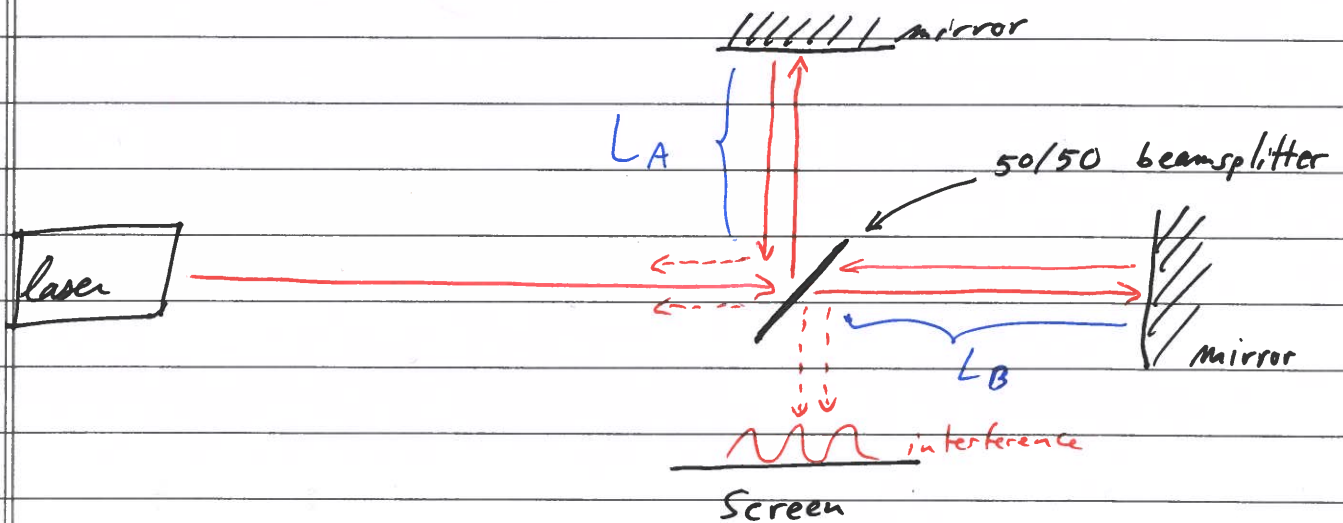


Thursday, September 4, 2025

## Coherence (continued)

### Review of the Michelson interferometer experiment



$$\Delta\phi = \phi_A - \phi_B = 2\pi \left[ 2(L_A - L_B) \bmod \lambda \right]$$

$$\begin{cases} \Delta\phi = 0 \Rightarrow \text{constructive interference} \\ \Delta\phi = \pi \Rightarrow \text{destructive interference} \end{cases}$$

Experimental Fact #1: For  $L_A \approx L_B$ , the fringe visibility is near 100%.  
i.e. roughly equal arms

Experimental Fact #2: For  $|L_A - L_B| \gg 0$ , the fringe visibility goes to zero.  
i.e. unbalanced arms

### Two explanations

$$(\Delta L = L_A - L_B)$$

#### 1) Bandwidth

Consider 2 optical frequencies  $f_1$  &  $f_2$  ( $\lambda_1 \neq \lambda_2$ )

$$\Delta\phi_1 = 2\pi \frac{\Delta L}{\lambda_1} = 2\pi \frac{\Delta L f_1}{c}$$

$$\lambda f = c$$

$$\frac{1}{\lambda} = \frac{f}{c}$$

$$\Delta\phi_2 = 2\pi \frac{\Delta L}{\lambda_2} = 2\pi \frac{\Delta L f_2}{c}$$

If  $\Delta\phi_1 \neq \Delta\phi_2$  differ by  $\pi$ , then the interference fringes of  $f_1$  &  $f_2$  anti-overlap  $\rightarrow$  overall interference will wash out.

$$\Delta\phi = \Delta\phi_1 - \Delta\phi_2 = 2\pi \frac{\Delta L}{c} \underbrace{(f_1 - f_2)}_{\Delta f}$$

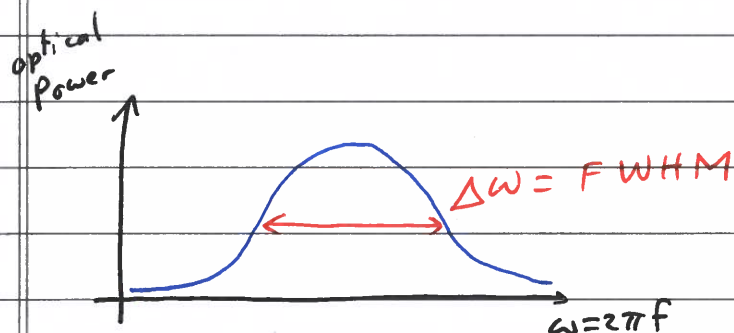
Example:  $\Delta\phi = \pi = 2\pi \frac{\Delta L}{c} \Delta f$

$$\Delta f = 1 \text{ GHz} \Rightarrow \Delta L = 0.15 \text{ m} = 15 \text{ cm}$$

$$\Delta f = 1 \text{ MHz} \Rightarrow \Delta L = 150 \text{ m}$$

$$\Delta f = \underbrace{1000 \text{ GHz}}_{\approx 1 \text{ nm @ } \lambda = 780 \text{ nm}} = 1 \text{ THz} \Rightarrow \Delta L = 0.15 \text{ mm} = 150 \text{ } \mu\text{m}$$

Definition: Coherence Length =  $L_c$  = characteristic difference in path length over which the interference pattern washes out.



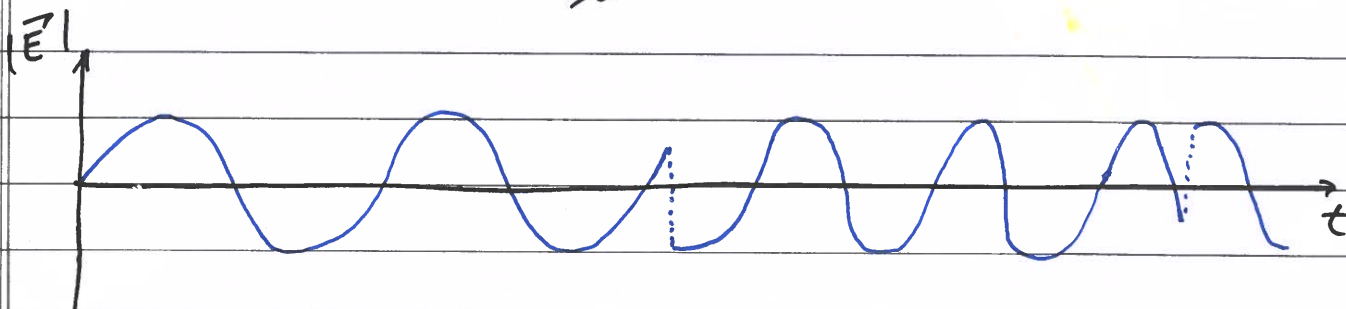
$$L_c \approx \frac{c}{\Delta\omega}$$

(depends on the exact nature of the source)

## 2) Dephasing time or coherence time

If the phase of the laser jumps (resets itself) on a characteristic time scale  $\tau_c$ , then the interference or wanders

washes out for  $\Delta L \gtrsim c\tau_c$



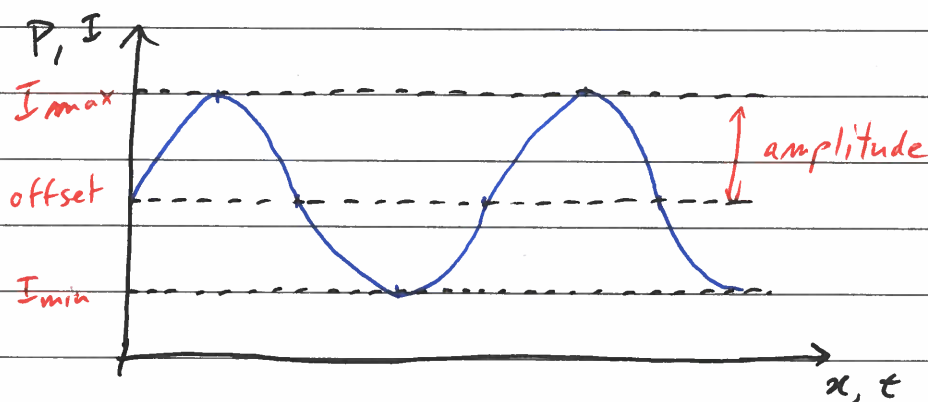
Which explanation is correct? Bandwidth or phase jumps?

↳ How could you design an experiment to distinguish them?

In fact, both explanations are equivalent, since in Fourier space (frequency space), phase jumps indicate that the frequency is not that well determined: In other words, the source has a certain bandwidth.

$$L_c = c\tau_c$$

Fringe visibility and  $g^{(1)}(\tau)$



Def: Fringe visibility =  $\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{\text{Amplitude of fringe}}{\text{Offset of fringe}}$

Fringe visibility = degree of first order coherence

$$\text{fringe visibility} = \begin{cases} = 0 & \rightarrow \text{no coherence} \\ = 1 & \rightarrow \text{perfect coherence} \end{cases}$$

1st order correlation function:

Consider the electric field due to light:  $E(t) = E_0 e^{-i\omega t + i\phi(t)}$

treat  $\vec{E}$ -field as a scalar for simplicity

(fixed position)  
 $\vec{r} = 0$

Def:  $g^{(1)}(\tau) = \frac{\langle E^*(t) E(t+\tau) \rangle_{\text{time}}}{\langle E^*(t) E(t) \rangle_{\text{time}}}$

$$= \frac{\langle E_0 e^{+i(\omega t - \phi(t))} E_0 e^{-i(\omega(t+\tau) - \phi(t+\tau))} \rangle_t}{\langle E_0 e^{+i(\omega t - \phi(t))} E_0 e^{-i(\omega t - \phi(t))} \rangle_t}$$

$$= \underbrace{e^{-i\omega\tau}}_{\uparrow} \langle e^{i(\phi(t+\tau) - \phi(t))} \rangle_t$$

often, this term is left out since it oscillates too quickly to be seen.

Theorem:  $|g^{(1)}(\tau)| = \text{fringe visibility}$  [for a 50/50 interferometer]

Corollary:  $g^{(1)}(\tau) = \text{degree of 1st order coherence}$

proof: Interferometer output =  $I = \langle |E(t) + E(t+\tau)|^2 \rangle_t$

$I_0 = E_0 E_0^*$

path A

path B

$$\Rightarrow I = \langle |E(t)|^2 \rangle + \langle |E(t+\tau)|^2 \rangle + \langle E(t) E^*(t+\tau) \rangle + \langle E^*(t) E(t+\tau) \rangle$$

$$= I_0 + I_0 + I_0 \langle e^{-i\omega t + i\phi(t)} e^{+i\omega(t+\tau) - i\phi(t+\tau)} \rangle + I_0 \langle e^{+i\omega t - i\phi(t)} e^{-i\omega(t+\tau) + i\phi(t+\tau)} \rangle$$

$$= I_0 \left( 2 + e^{i\omega\tau} \underbrace{e^{-i(\phi(t+\tau) - \phi(t))}}_{Ae^{-i\alpha}} + e^{-i\omega\tau} \underbrace{e^{i(\phi(t+\tau) - \phi(t))}}_{Ae^{i\alpha}} \right)$$

$$= 2 I_0 \left( 1 + |g^{(1)}(\tau)| \cos(\omega\tau - \alpha) \right)$$

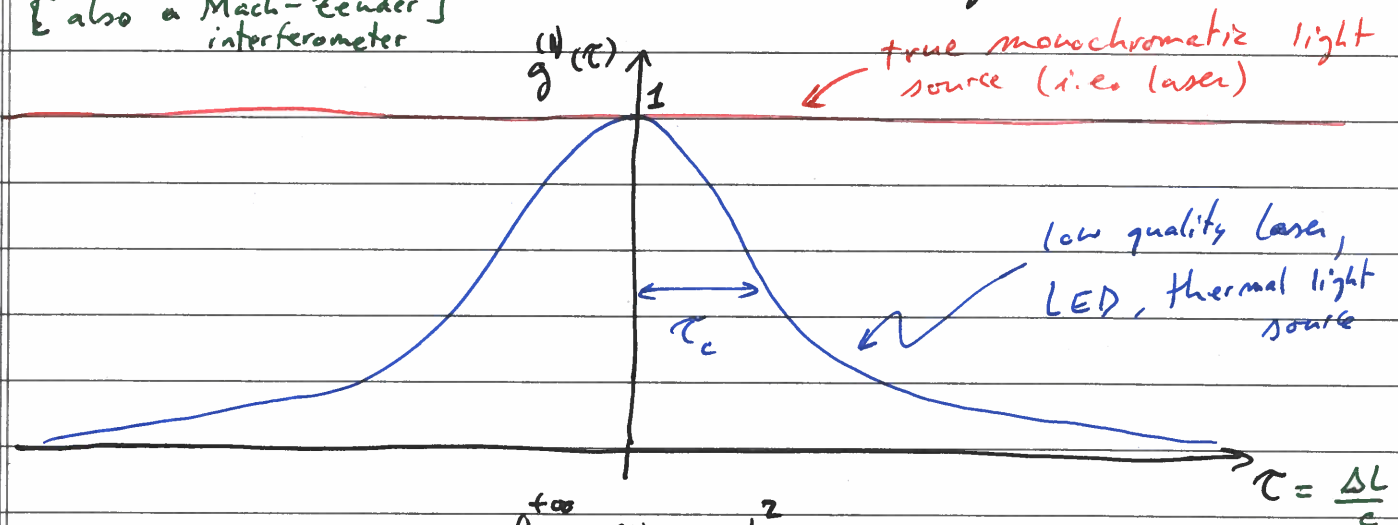
Note 1:  $|g^{(1)}(\tau)| = 1 \Rightarrow$  coherent light

$|g^{(1)}(\tau)| = 0 \Rightarrow$  completely incoherent light

Note 2:  $g^{(1)}(0) = 1$  always

$\hookrightarrow$  white light interferometer requires  $\tau = 0$   
 $\hookrightarrow \Delta L \approx 0$

A Michelson interferometer measures  $g^{(1)}(\tau)$   
 [also a Mach-Zehnder interferometer]



Definition:  $\tau_c = \int_{-\infty}^{+\infty} |g^{(1)}(\tau)|^2 d\tau = \text{coherence time}$