Thursday, November 20, 2025

Magnetic Traps

(I) Hamiltonian

The Hamiltonian for an atom in an external B-field is

momentum of atom

$$H = \frac{P_{cm}^2}{2M} + \underbrace{H_{atom}}_{\frac{p^2}{2m} + \frac{q^2}{R}} + \underbrace{H_{Zeeman}}_{= -\vec{\mu}\cdot\vec{B} = \mu_B g_F m_F |\vec{B}|}$$

mass of atom

"mass of e^-"

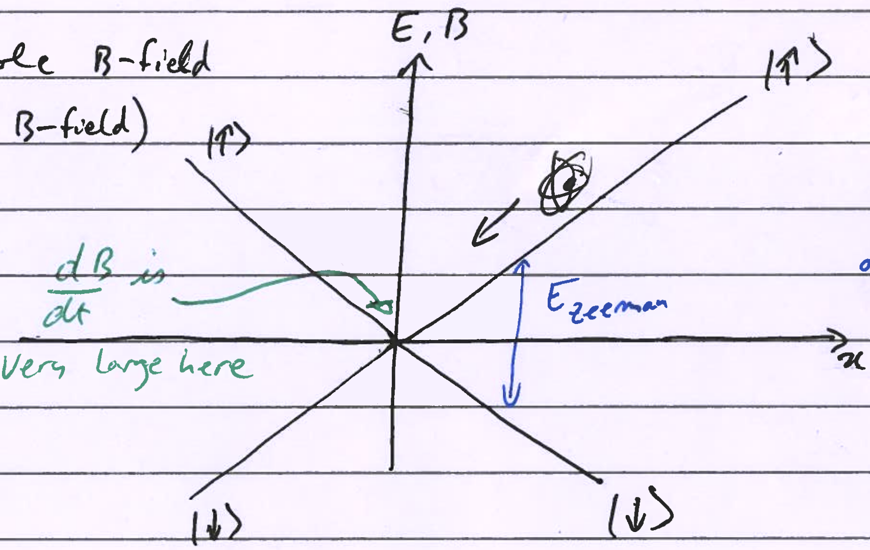
quantization axis given by the direction of the local magnetic field.

Note: Internal energy level shift = potential energy of atom

If the atom moves "slowly" (adiabatically) in the magnetic field, then the "angle" between $\vec{\mu}$ & $\vec{B}$ stays the same, and the atom stays in its m_F-level/state defined by the local magnetic field quantization axis.

Semiclassical criterion for adiabatic motion: $\dfrac{H_{Zeeman}}{\hbar} \gg \dfrac{1}{B}\left(\dfrac{dB}{dt}\right)$

Ex: Quadrupole B-field (linear B-field)



You don't want the Fourier frequency components of frequency/energy ramp to include ω_{Zeeman} $[E_{Zeeman} = \hbar\omega_{Zeeman}]$

$$H_{Zeeman} = -\vec{\mu}\cdot\vec{B} = -|\vec{\mu}|\,|\vec{B}|\,\underbrace{\cos\theta}_{\text{constant if motion is adiabatic}}$$ ← classical expression

positive

$$= \underbrace{m_F \, g_F \, \mu_B}_{\text{constant}} \, |\vec{B}|$$ ← quantum expression

constant if motion is adiabatic

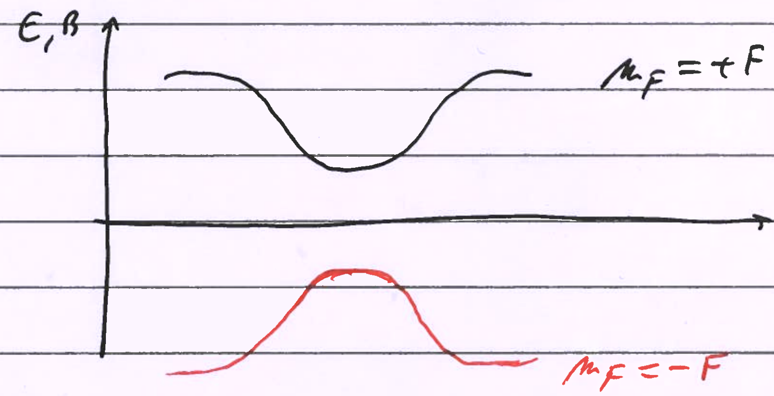
Magnitude of $|\vec{B}|$ determines the potential energy

with $\mu_B = \dfrac{|e|\hbar}{2m_e} = 9.27408\times10^{-24}\ J/T$

$$= h \times 1.4\ MHz/G$$

$\Rightarrow$ $\boxed{\text{Magnetic potential energy is proportional to } |\vec{B}|}$

<u>Note</u>: If $m_F = +F$ is trapped, then $m_F = -F$ is anti-trapped



(II) Earnshaw's theorem for magnetic fields

[Electrostatic version proved in 1842, magnetostatic case proved and rediscovered in 1980s]

There are no static magnetic field maxima in current free regions of space.

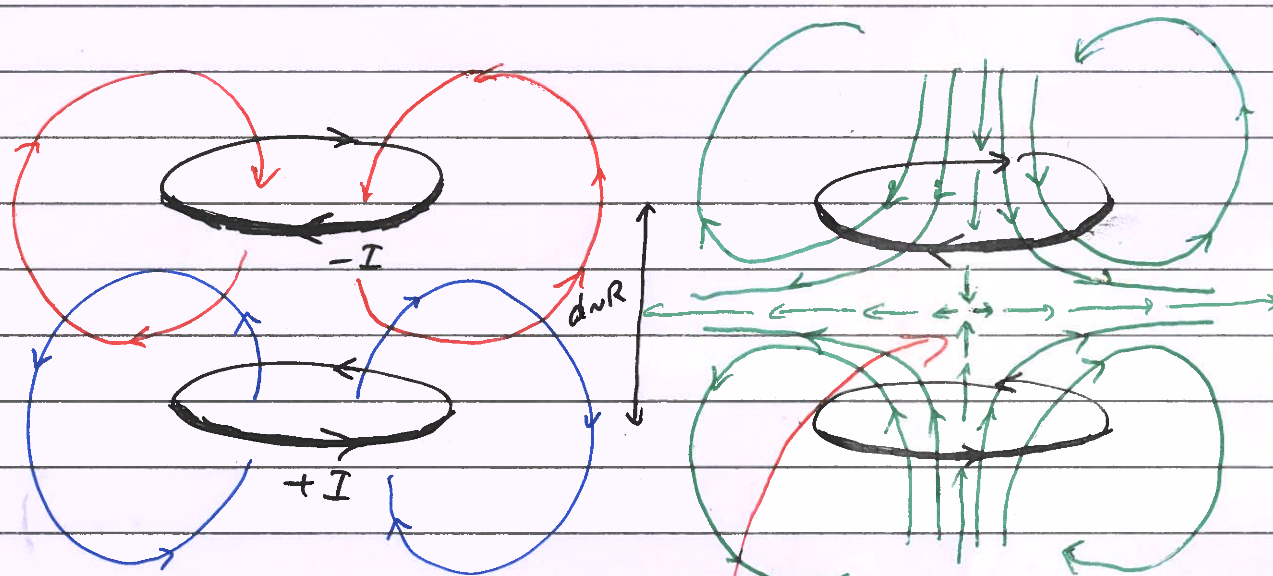
[Magnetic field minima are possible]

Corollary: states with $m_F g_F < 0$ (i.e. $m_s = \downarrow$) **cannot** be magnetically trapped.

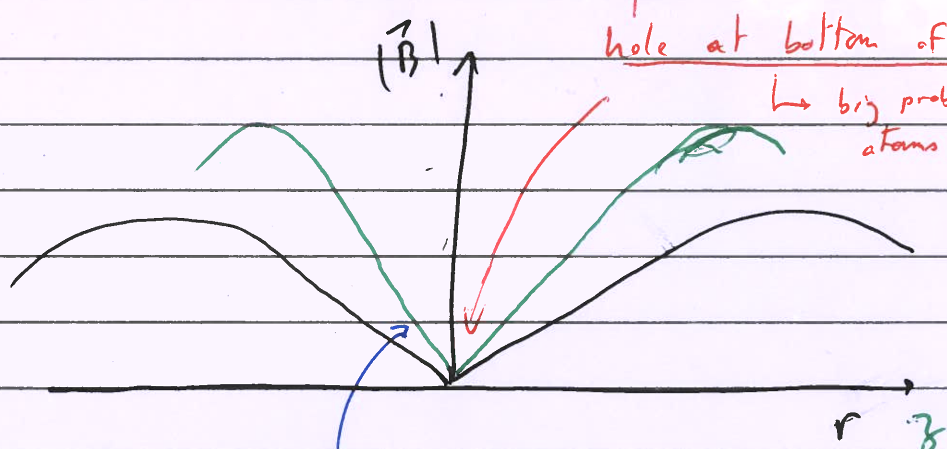
Take home message:

- states with $m_F g_F > 0$ are low magnetic field seekers
  - ↳ trappable
- states with $m_F g_F < 0$ are high magnetic field seekers.
  - ↳ **not** trappable

III) Quadrupole Trap (anti-Helmholtz coil trap)



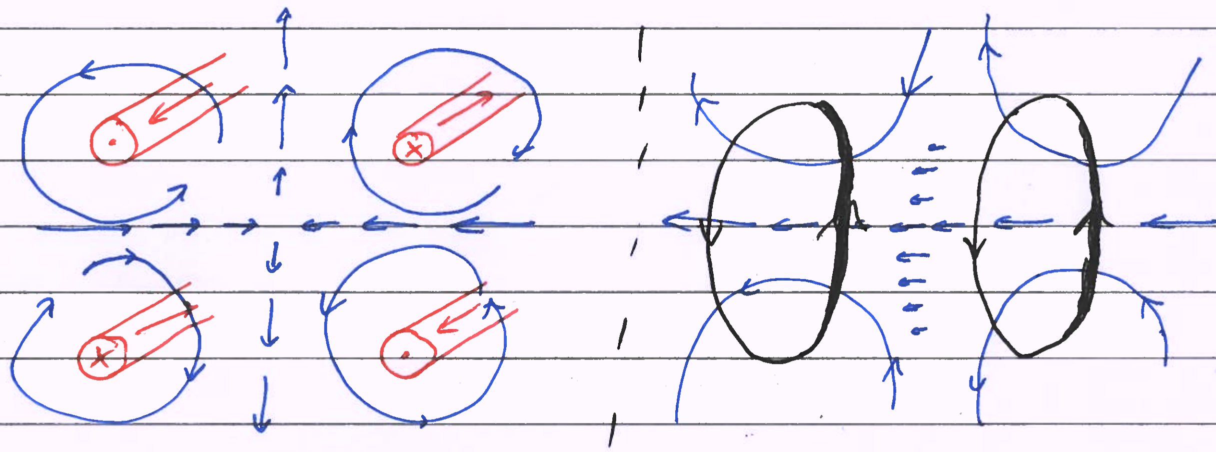
hole at bottom of trap!!
↳ big problem when atoms are very cold
"Majorana loss"



$\text{slope}|_z = 2\ \text{slope}|_r$

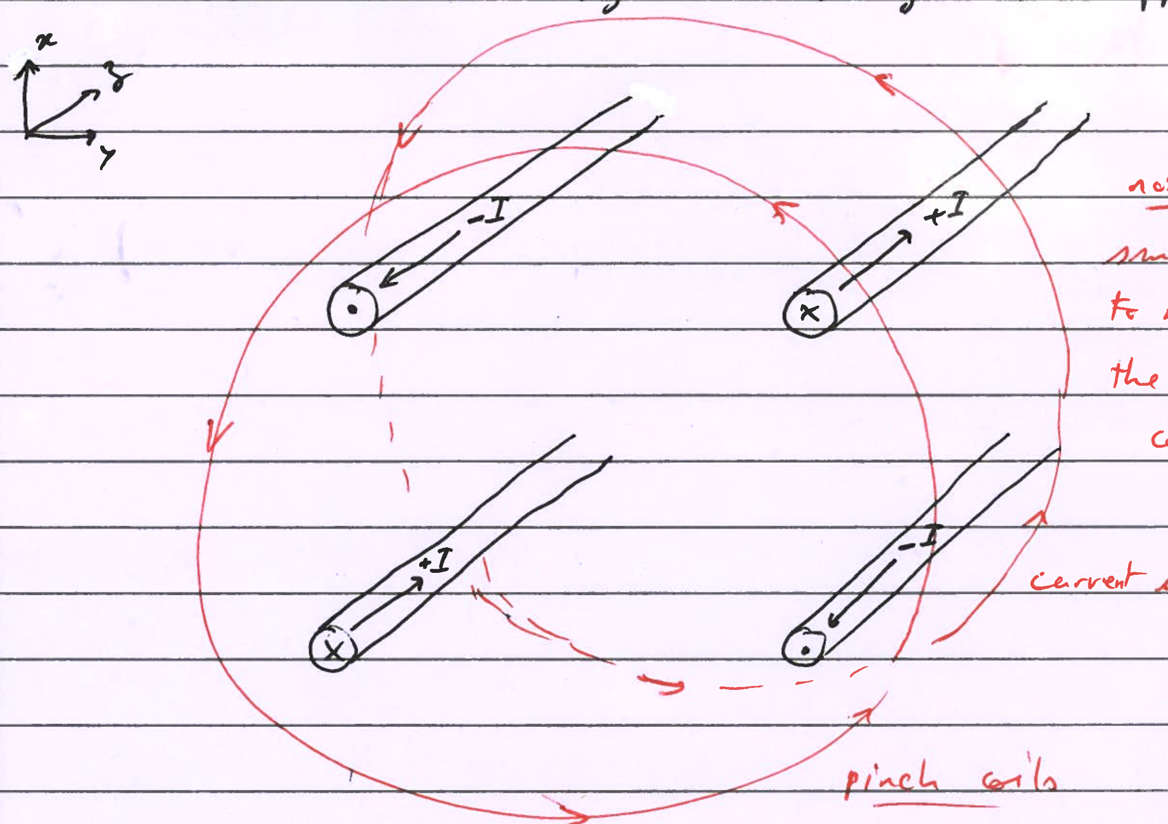
IV) Ioffe - Pritchard Trap

Consider the current arrangements



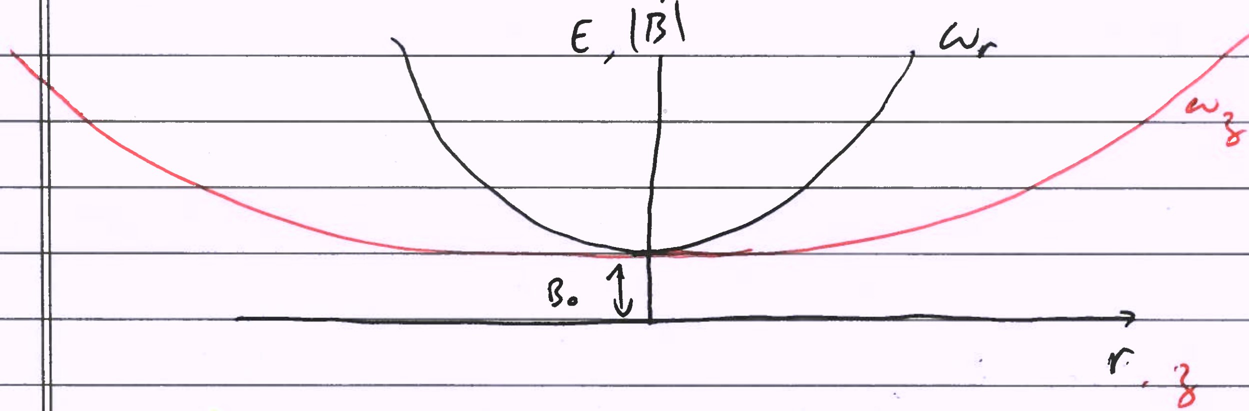
⇒ local minimum (zero) in plane | Local minimum along main axis, but not along radial directions. (saddle point)

Combine both current configurations to get 3D trapping



note: i is small enough to not disrupt the radial confinement

The Ioffe - Pritchard potential is harmonic in all 3 dimensions



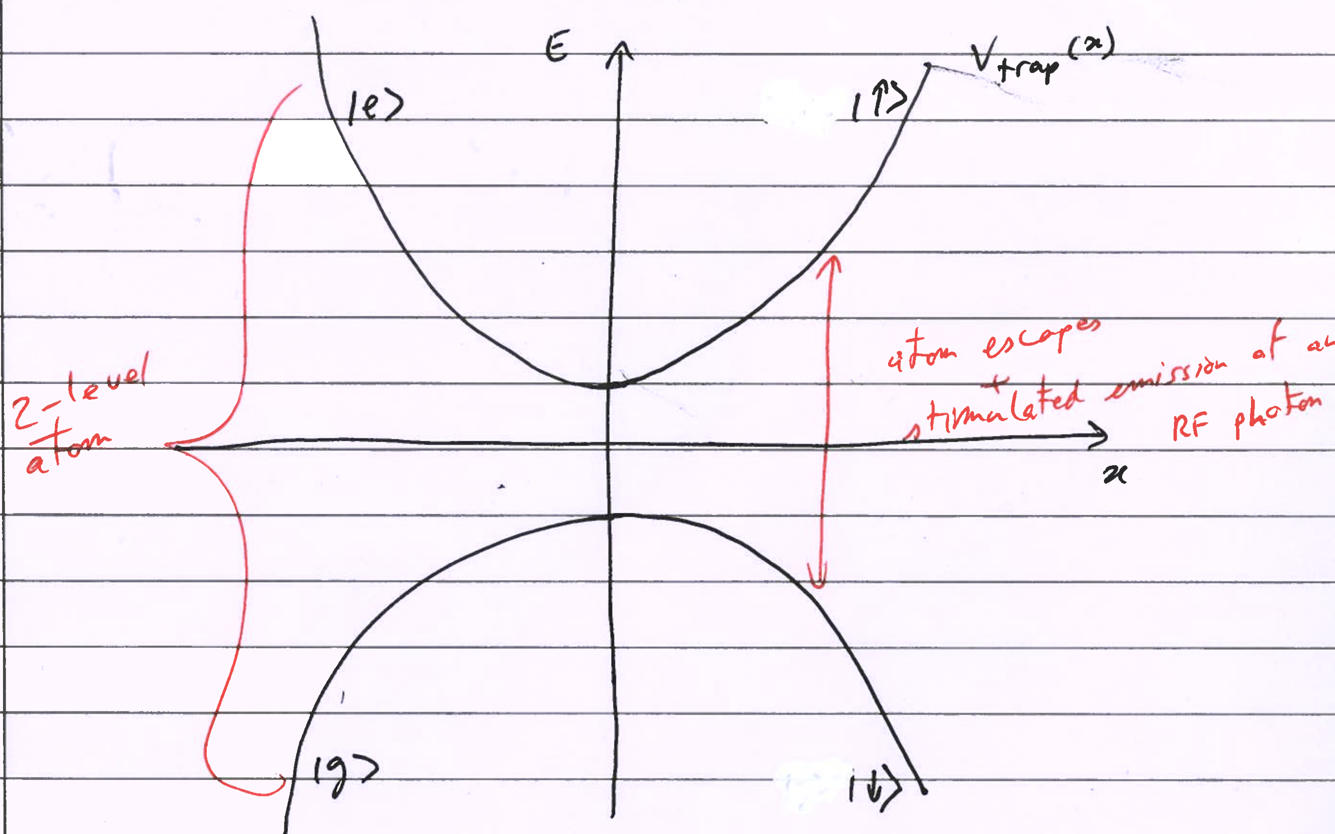
For $\omega_z \ll \omega_r \Rightarrow$ cigar-shaped trap

$$H_{I.P} = \frac{1}{2} m \omega_r^2 r^2 + \frac{1}{2} m \omega_z^2 z^2 = V_{\text{Ioffe-Pritchard}}$$

show PowerPoint slides on RF evaporation

(V) Dressed atom picture of evaporation (1D, spin-½)

Assume a harmonic trapping potential: $V_{trap}(x)$



$$
\begin{array}{c c}
 & \begin{array}{cc} |g\rangle & |e\rangle \end{array} \\
\begin{array}{c} \langle g| \\ \langle e| \end{array} & H = \begin{pmatrix} -V_{trap}(x) & 0 \\ 0 & +V_{trap}(x) \end{pmatrix} + \underbrace{\hbar\omega_{rf}\begin{pmatrix} N+1 & 0 \\ 0 & N \end{pmatrix}}_{\text{reset energy offset: } \hbar\omega_{rf}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}} + \hbar\begin{pmatrix} 0 & \Omega/2 \\ \Omega^*/2 & 0 \end{pmatrix}
\end{array}
$$

$$\Rightarrow H = \begin{pmatrix} \hbar\omega_{rf} - V_{trap}(x) & \hbar\Omega/2 \\ \hbar\Omega^*/2 & +V_{trap}(x) \end{pmatrix}$$

let's calculate the eigenenergies: $\det(H - \lambda I) = 0$

$$\Rightarrow \begin{vmatrix} \hbar\omega_{rf} - V_{trap}(x) - \lambda & \hbar\Omega/2 \\ \hbar\Omega^*/2 & V_{trap}(x) - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow \left(\hbar\omega_{rf} - V_{trap}(x) - \lambda\right)\left(V_{trap}(x) - \lambda\right) - \frac{\hbar^2|\Omega|^2}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \left(\hbar\omega_{rf} - \cancel{V_{trap}(x)} + \cancel{V_{trap}(x)}\right)\lambda + \left(\hbar\omega_{rf} - V_{trap}(x)\right)V_{trap}(x) - \frac{\hbar^2|\Omega|^2}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \hbar\omega_{rf}\lambda + \left(\hbar\omega_{rf} - V_{trap}\right)V_{trap} - \frac{\hbar^2|\Omega|^2}{4} = 0$$

thus $E_\pm = \lambda_\pm = \frac{\hbar\omega_{rf}}{2} \pm \frac{1}{2}\sqrt{\underbrace{\hbar^2\omega_{rf}^2 - 4\left(\hbar\omega_{rf} - V_{trap}\right)V_{trap}}_{\hbar^2\omega_{rf}^2 - 4\hbar\omega_{rf}V_{trap} + 4V_{trap} = \underbrace{\left(\hbar\omega_{rf} - 2V_{trap}\right)^2}_{\text{"detuning"}}} + \hbar^2|\Omega|^2}$

$$\Rightarrow \boxed{E_\pm = \frac{\hbar\omega_{rf}}{2} \pm \frac{1}{2}\sqrt{\left(\hbar\omega_{rf} - 2V_{trap}(x)\right)^2 + \hbar^2|\Omega|^2}}$$