PHYS 430 and 631: Quantum Optics & Atomics
Practice problems

The Quantized Electromagnetic Field

1. Fock states

Consider the Fock state $|n\rangle_{k,s}$ with *n* excitations of the photon field with momentum ***k*** and polarization *s* in a volume V (in vacuum).

Compute the following quantities:

a) Average electric field: $\langle \vec{E} \rangle = {}_{k,s}\langle n|\vec{E}|n\rangle_{k,s}$

b) Variance of the electric field: $\Delta\vec{E}^2 = {}_{k,s}\langle n|\vec{E}^2|n\rangle_{k,s} - \left({}_{k,s}\langle n|\vec{E}|n\rangle_{k,s}\right)^2$

c) Average photon number $\langle N \rangle = {}_{k,s}\langle n|N|n\rangle_{k,s}$

and photon number variance $\Delta N^2 = {}_{k,s}\langle n|N^2|n\rangle_{k,s} - \left({}_{k,s}\langle n|N|n\rangle_{k,s}\right)^2$

2. Coherent states I

Consider the coherent state $|\alpha\rangle_{k,s} = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_{k,s}$, where α is a complex number and $|n\rangle_{k,s}$ are the photon Fock states described in problem 1.

Compute the following quantities:

a) Average electric field: $\langle \vec{E} \rangle = {}_{k,s}\langle \alpha|\vec{E}|\alpha\rangle_{k,s}$

b) Variance of the electric field: $\Delta\vec{E}^2 = {}_{k,s}\langle \alpha|\vec{E}^2|\alpha\rangle_{k,s} - \left({}_{k,s}\langle \alpha|\vec{E}|\alpha\rangle_{k,s}\right)^2$

c) Average photon number $\langle N \rangle = {}_{k,s}\langle \alpha|N|\alpha\rangle_{k,s}$

and photon number variance $\Delta N^2 = {}_{k,s}\langle \alpha|N^2|\alpha\rangle_{k,s} - \left({}_{k,s}\langle \alpha|N|\alpha\rangle_{k,s}\right)^2$

d) For α=10, calculate the mean photon number, the variance in the photon number,and the quantum uncertainty in the optical phase.

3. Coherent States II

Consider two coherent states |α⟩ and |β⟩. Show that they are not orthogonal by proving the relation:

$$\langle \alpha|\beta\rangle = \exp\left[-\frac{1}{2}\left(|\alpha|^2 + |\beta|^2 - 2\alpha^*\beta\right)\right]$$