Thursday, September 11, 2025 #1

recall: $2^{nd}$ order auto-correlation function: $g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau)\rangle}{\langle I(t)\rangle^2}$

## Chaotic light sources (e.g. thermal light source)

**Example:** Monochromatic light with random time-dependent phases generated by $N$ atoms (disregard polarization).

atom $j$ emits: $E_j(t) = E_0 e^{-i\omega t + i\phi_j(t)}$

$\phi_j(t)$ changes randomly in the range $[0, 2\pi[$ over a characteristic time $t = \tau_c$ (coherence time)

For $N$ atoms: $E_{total} = \sum_{j=1}^{N} E_j(t)$

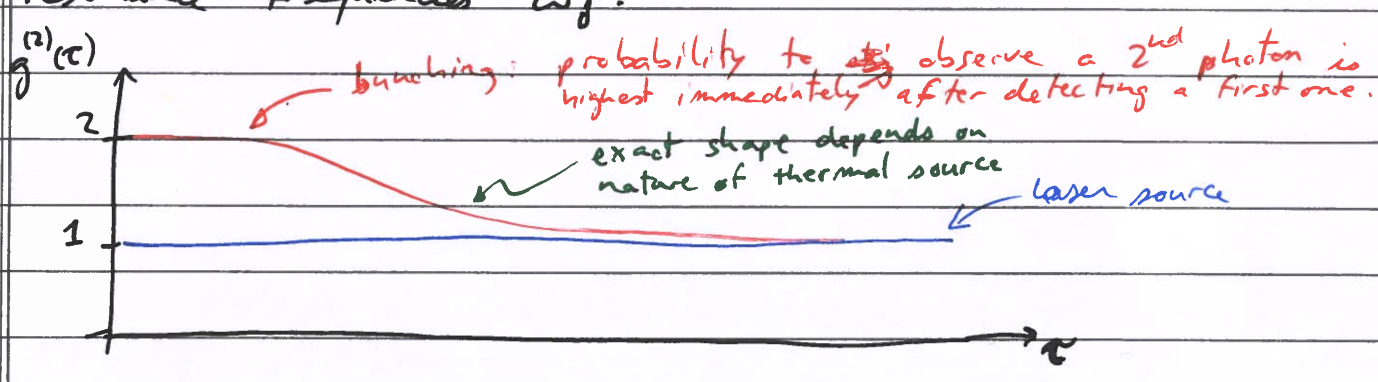
$$= E_0 e^{-i\omega t} \sum_{j=1}^{N} e^{i\phi_j(t)}$$

$$\Rightarrow \langle I_{total}(t)\rangle_t = \underset{\text{some algebra math}}{\longrightarrow} \dots = \langle I_{\substack{\text{single}\\\text{atom}}}\rangle_t N$$

One can show that $\boxed{g^{(2)}(\tau=0) = 2}$ and $\boxed{g^{(2)}(\tau \gg \tau_c) = 1}$

[see problem set #3]

The same results are true for chaotic thermal light in which $\phi_j(t) = cst_j$, but there is a spread of atomic resonance frequencies $\omega_j$.



For chaotic light (thermal light), one can show that

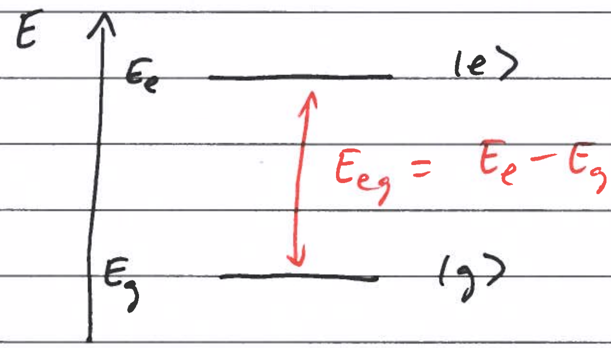
$$g^{(2)}(\tau) = 1 + \left|g^{(1)}(\tau)\right|^2$$

For an arbitrary classical light source, the following properties hold:

- $g^{(2)}(\tau) = g^{(2)}(-\tau)$
- $1 \leq g^{(2)}(0) \leq \infty$ and $0 \leq g^{(2)}(\tau > 0) \leq \infty$
- $g^{(2)}(\tau) \leq g^{(2)}(0)$ ← not true for "quantum light"

## 2-level atoms

(I) **Definition**: A 2-level atom is a quantum mechanical system with 2 distinct eigenstates & eigenenergies.




The basic Hamiltonian for the system is

$$H_0 = \begin{array}{c c} & \begin{array}{cc} |g\rangle & \quad |e\rangle \end{array} \\ \begin{array}{c} \langle g| \\ \langle e| \end{array} & \begin{bmatrix} E_g & 0 \\ 0 & E_e \end{bmatrix} \end{array}$$

One can write an ~~a~~ arbitrary state of the system as

$$|\psi\rangle = \alpha|g\rangle + \beta|e\rangle \equiv |\alpha||g\rangle + |\beta|e^{i\phi}|e\rangle$$

normalization requires

$$|\langle\psi|\psi\rangle|^2 = |\alpha|^2 + |\beta|^2 = 1$$

$$\Rightarrow \quad |\psi\rangle \equiv \cos\theta\,|g\rangle + \sin\theta\, e^{i\phi}|e\rangle$$

where $\tan\theta = \dfrac{|\beta|}{|\alpha|}$

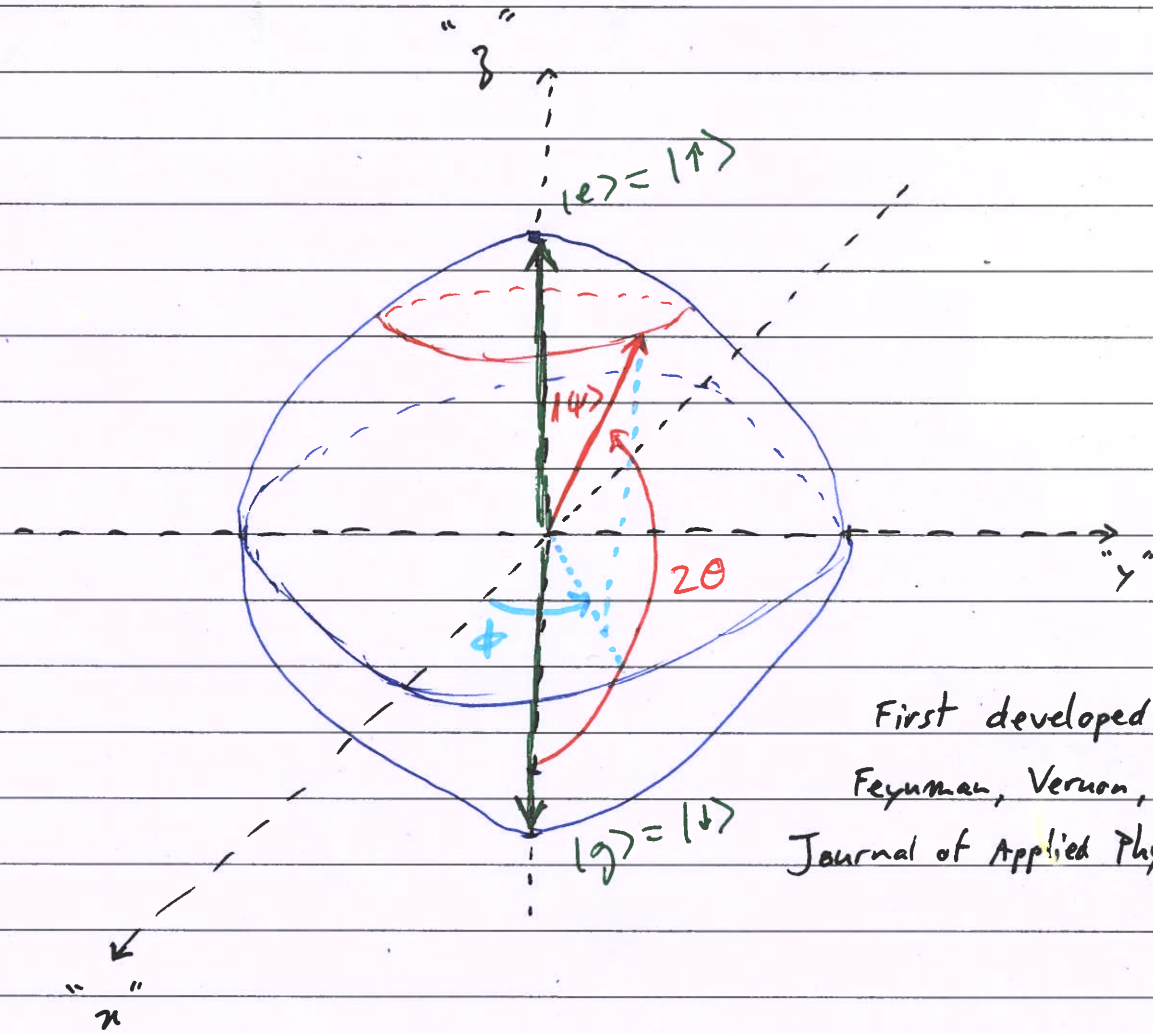
## II Bloch Sphere picture

A 2-level system is like a spin-$\frac{1}{2}$ system with

$$|\uparrow\rangle = |e\rangle \quad \text{and} \quad |\downarrow\rangle = |g\rangle$$

Thus
$$|\psi\rangle = \cos\theta\,|g\rangle + e^{i\phi}\sin\theta\,|e\rangle$$
$$= \cos\theta\,|\downarrow\rangle + e^{i\phi}\sin\theta\,|\downarrow\rangle$$

with
$$\begin{cases} 0 \leq \phi < 2\pi \\ 0 \leq \theta \leq \pi/2 \end{cases}$$




First developed by Feynman, Vernon, and Hellwarth Journal of Applied Physics 28, 49 (1957)

## III Time-dependence

What happens when you let an arbitrary state evolve in time?

Easy: $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle \Rightarrow |\psi(t)\rangle = \alpha e^{-i\frac{E_g t}{\hbar}}|g\rangle + \beta e^{-i\frac{E_e t}{\hbar}}|e\rangle$

$$= \underbrace{e^{-i\frac{E_g t}{\hbar}}}_{\text{overall phase}} \left[\alpha|g\rangle + \beta \underbrace{e^{-i\frac{(E_e - E_g)t}{\hbar}}}_{\text{relative phase}}|e\rangle\right]$$

overall phase ↳ not important

relative phase ↳ totally important

$$P(|g\rangle)_{|\psi(t)\rangle} = |\langle g|\psi(t)\rangle|^2 = |\alpha|^2$$

$$P(|e\rangle)_{|\psi(t)\rangle} = |\langle e|\psi(t)\rangle|^2 = |\beta|^2$$

no measureable time-dependence

[Show phase precession on Bloch sphere]

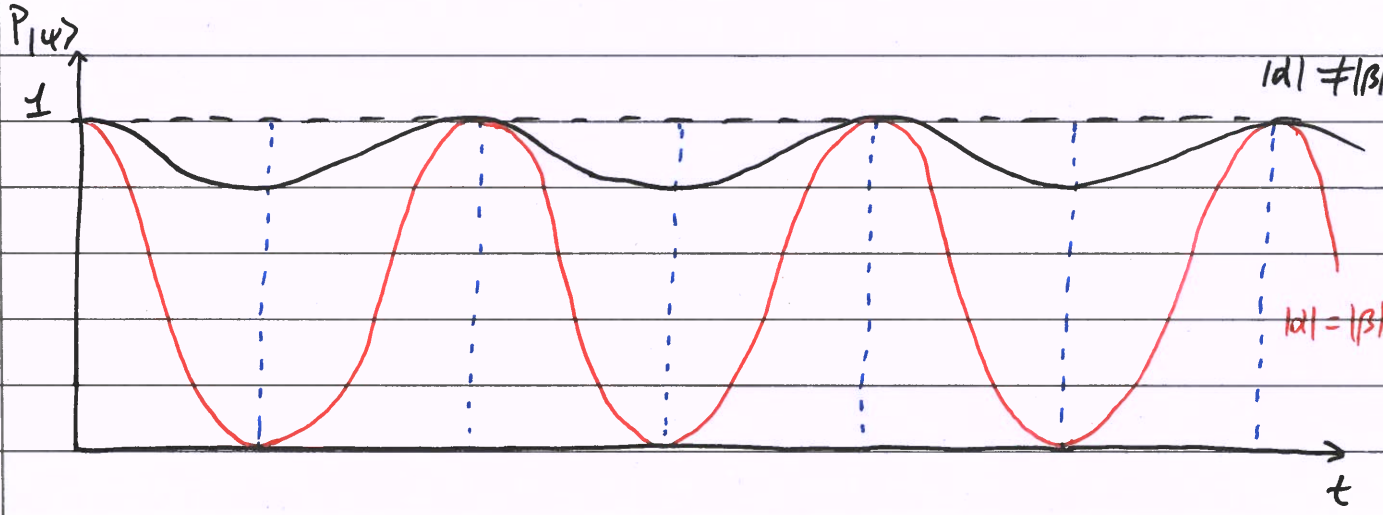
more interesting

$$P(|\psi\rangle)_{|\psi(t)\rangle} = |\langle\psi|\psi(t)\rangle|^2$$

$$= \left|\left(\alpha^*\langle g| + \beta^*\langle e|\right)\left(\alpha e^{-i\frac{E_g t}{\hbar}}|g\rangle + \beta e^{-i\frac{E_e t}{\hbar}}|e\rangle\right)\right|^2$$

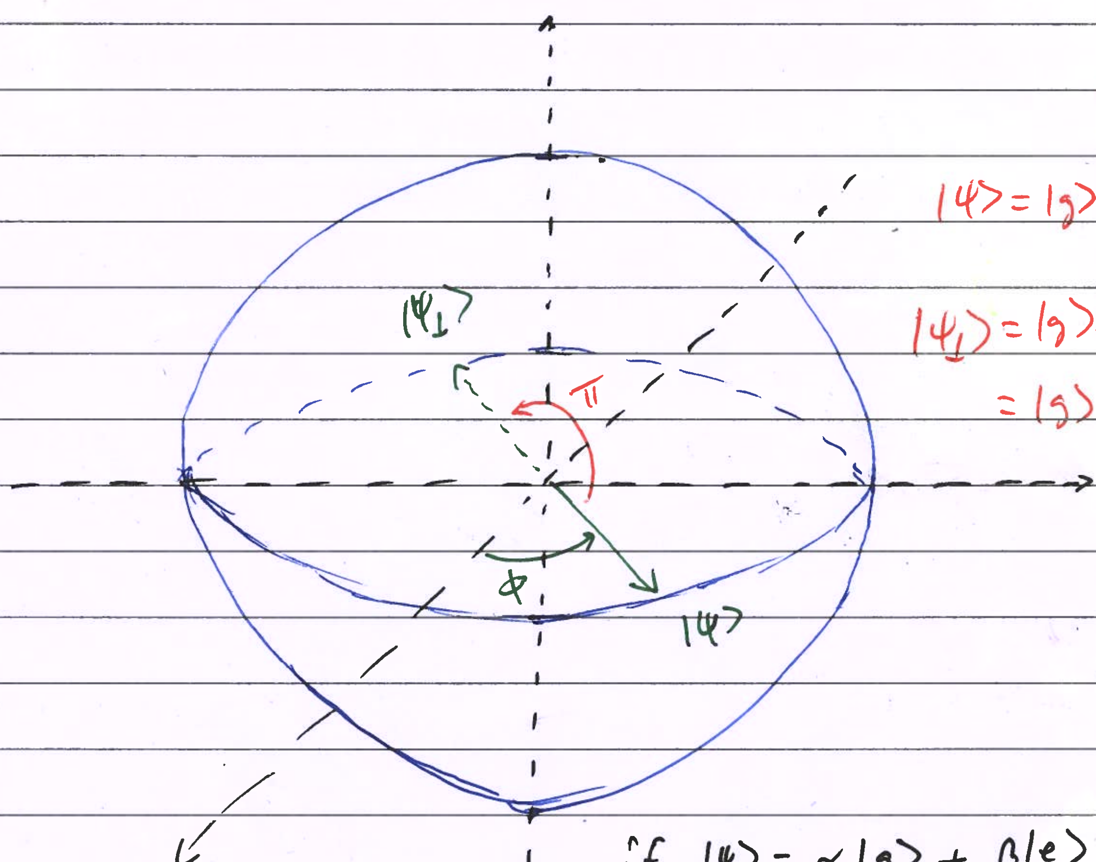
(some algebra)

$$= \frac{1}{2}\left[1 + \left(|\alpha|^2 - |\beta|^2\right)^2\right] + \frac{1}{2}\left[1 - \left(|\alpha|^2 - |\beta|^2\right)^2\right]\cos(\underbrace{\omega_{eg}}_{E_{eg}/\hbar} t)$$




So there is time dependence if you look in another basis!!

## Bloch Sphere picture




$|\psi\rangle = |g\rangle + e^{i\phi}|e\rangle$

$|\psi_\perp\rangle = |g\rangle + e^{i(\phi-\pi)}|e\rangle$

$= |g\rangle - e^{i\phi}|e\rangle$

if $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$

then $|\psi_\perp\rangle = \beta^*|g\rangle - \alpha^*|e\rangle$