[Start of class: - 5 minute demo of optical tweezers by Bjorg Larson] #1
- 5 minute demo of HeNe Michelson interferometer
+ LIGO + atom interferometer discussion]

Thursday, September 25, 2025

The Bloch Sphere Revisited

In the dressed atom picture for $\Omega \in \mathbb{R}$, the Hamiltonian is given by:

$$H = \hbar \begin{array}{c} \begin{array}{cc} |g,N+1\rangle & |e,N\rangle \end{array} \\ \begin{bmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{bmatrix} \end{array}$$

(subtracted $\hbar\,\delta/2$ as an energy offset)

define: $|\psi(t)\rangle = C_g(t)\,|g,N+1\rangle + C_e(t)\,|e,N\rangle$

Schrodinger Equation:

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$\Rightarrow \quad i\hbar \frac{d}{dt}\begin{pmatrix} C_g(t) \\ C_e(t) \end{pmatrix} = \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}\begin{pmatrix} C_g(t) \\ C_e(t) \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} i\dfrac{d}{dt}C_g(t) = \delta/2\; C_g(t) + \Omega/2\; C_e(t) \\[2ex] i\dfrac{d}{dt}C_e(t) = \dfrac{\Omega}{2}\, C_g(t) - \delta/2\; C_e(t) \end{cases}$$

Feynman et al., Journal of Applied Physics 28, 49 (1957), discovered that with the substitution

$$R_1 = c_g c_e^* + c_g^* c_e$$
$$R_2 = i\,(c_g c_e^* - c_g^* c_e)$$
$$R_3 = |c_g|^2 - |c_e|^2$$

$\Rightarrow \vec{R} = (R_1, R_2, R_3)$ = "state vector"

the time evolution (i.e. Schrodinger eqn) is described by

$$\boxed{\frac{d\vec{R}}{dt} = \vec{\Omega} \times \vec{R}}$$

where $\vec{\Omega} = (\Omega, 0, \delta)$ = "Rabi vector"

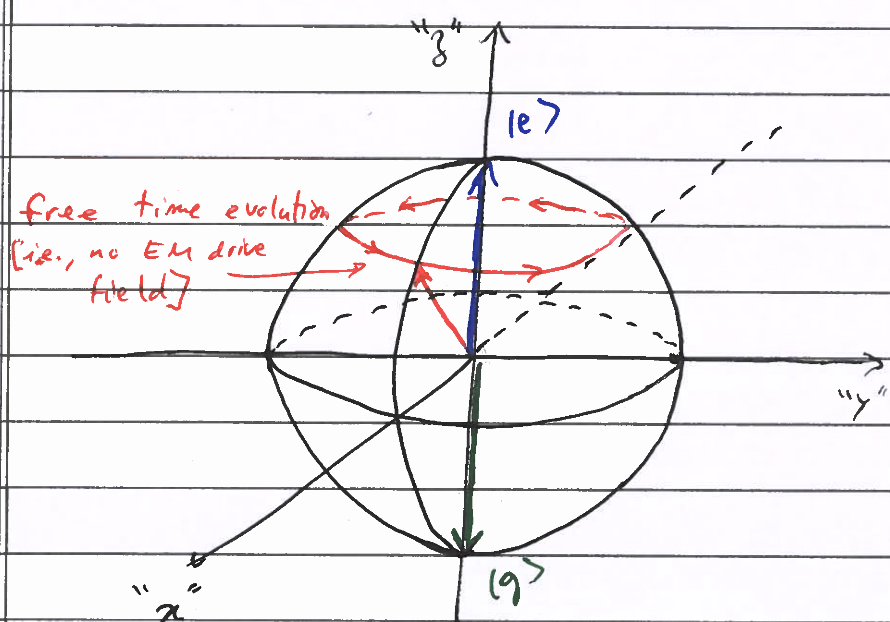
Equation of motion for circular motion of $\vec{R}$, where $\vec{\Omega}$ is the "angular velocity".

<u>note</u>: $|\vec{\Omega}| = \Omega' = \sqrt{\Omega^2 + \delta^2}$ = generalized Rabi freq.

<u>note 1</u>: $|\vec{R}|^2 = R_1^2 + R_2^2 + R_3^2 = 1 \Rightarrow \vec{R}$ "lives" on the surface of a sphere.

<u>note 2</u>: $\left|\frac{d}{dt}\vec{R}\right| = cst \Rightarrow$ constant circular motion.

<u>Reminder</u>: <u>"old/original" Bloch sphere picture</u>

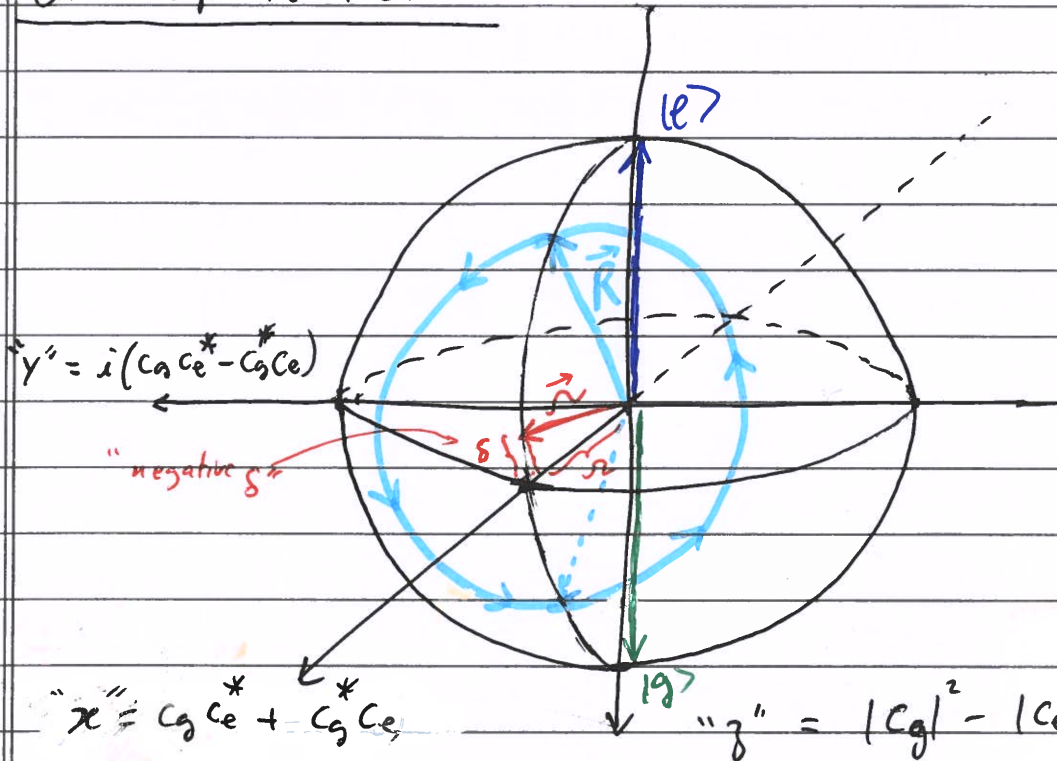


remove the free time evolution + δ precession

= remove precession a laser frequency

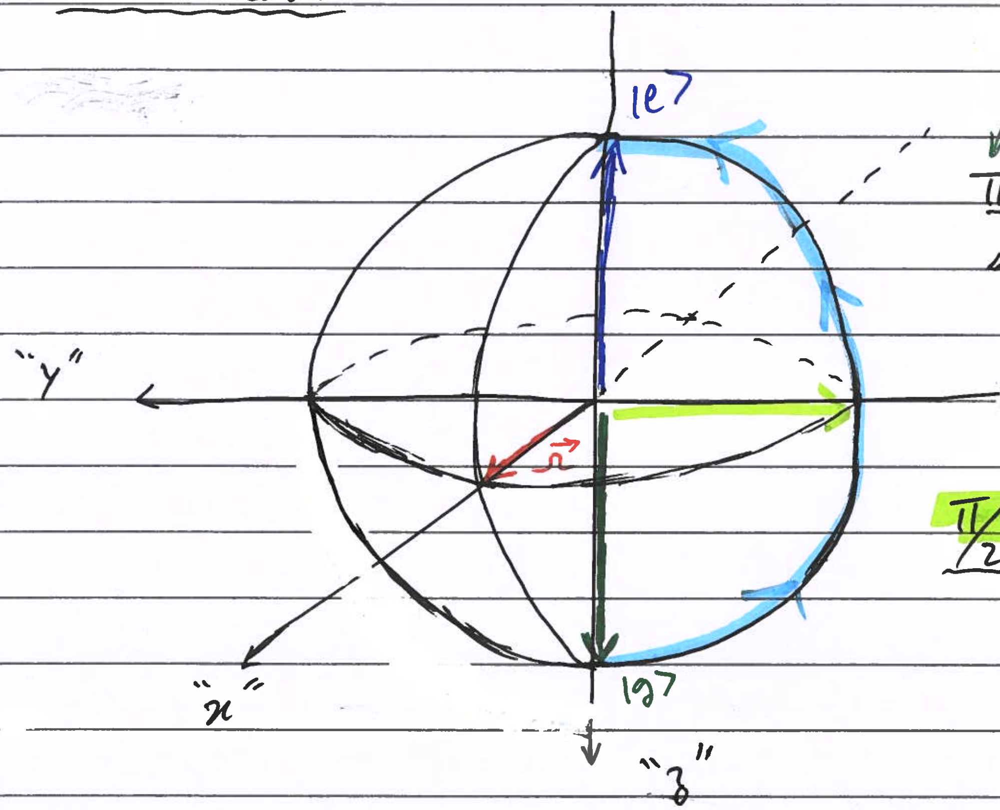
= go to the rotating frame at the laser frequency

Bloch Sphere Picture



$\delta = 0$ case

Δt = duration of EM pulse



π-pulse: Δt chosen such that $|g\rangle \longrightarrow |e\rangle$

$\pi/2$-pulse: Δt chosen such that $|g\rangle \rightarrow \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$

$\Omega \Delta t$ = pulse area

↳ determines "rotation angle"

see Sept. 16 lecture : eqn. 6a & 6b for $\delta = 0$

$$|\psi(t)\rangle = c_g(t)\,|g\rangle + c_e(t)\,|e\rangle \qquad (\text{in rotating frame})$$

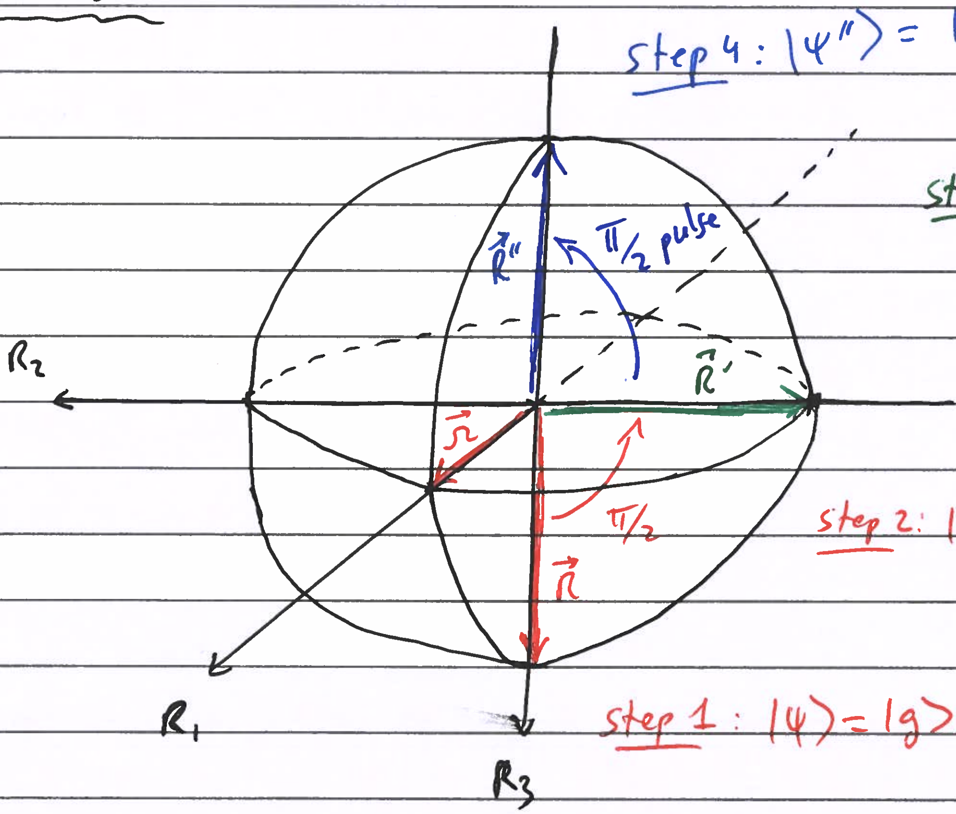
$$= \cos\left(\frac{\Omega t}{2}\right)|g\rangle - i \sin\left(\frac{\Omega t}{2}\right)|e\rangle$$

pick $\frac{\Omega t}{2} = \begin{cases} \pi \\ \text{or} \\ \pi/2 \end{cases}$

Application : Atomic clocks / Ramsey interferometer

Invented by Norman Ramsey (Nobel 1989)
(using the "method of separated oscillatory fields" developed in 1949)

Suppose $\delta = 0$



step 4 : $|\psi''\rangle = |e\rangle$

step 3 : wait time Δt counted with same oscillator used for $\pi/2$ pulse

step 2 : $|\psi'\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$

$= \frac{1}{\sqrt{2}}|g\rangle + \frac{e^{-i\phi}}{\sqrt{2}}|e\rangle$

$\phi = \pi/2$

step 1 : $|\psi\rangle = |g\rangle$