Physics 430 and 631: Quantum Optics & Atomics
Due date: Thursday, October 2, 2025

Problem Set #4

Dressed Atoms and Optical Trapping

Problem #1: Optical trapping

Part 1: Write a 2-page essay (typed, double spaced) that summarizes the related results of the two papers:

A. Ashkin, "Acceleration and Trapping of Particles by Radiation Pressure", *Phys. Rev. Lett.* **24**, 156 (1970).

S. Chu, J. E. Borkholm, A. Ashkin, and A. Cable, "Experimental Observation of Optically Trapped Atoms", *Phys. Rev. Lett.* **57**, 314 (1986).

Part 2: On a separate sheet, provide a quantitative justification for one of the quantitative claims in one of the two papers above … *hint: pick a very simple claim, since most of the quantitative claims will be very difficult to prove.*

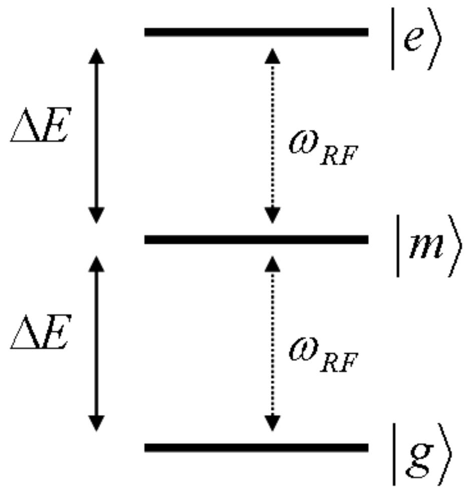
Problem #2: Mollow triplet at low photon number

a) Calculate the splitting of the Mollow triplet for ^{87}Rb for a laser a beam with 0 photons/s, 1 photon/s, 2 photons/s, 3 photons/s. The laser has a flat-top intensity profile with a diameter of 100 μm at a wavelength of 780 nm.

b) Repeat the above calculations, but in the case that the atoms are in an optical cavity which causes the laser beam to be reflected upon itself 10^6 times.

Problem #3: 3-Level Rabi Flopping

Solve the degenerate 3-level Rabi flopping problem (similar to the 5-level Rabi flopping movie and data that I showed in class, except easier). Consider a 3-level atom with states $|g\rangle$, $|m\rangle$, and $|e\rangle$ with the following energy level structure:



An RF magnetic field with frequency ω_{RF} is applied to the atom resulting in a (resonant) 2-level Rabi frequency of Ω for both the $|g\rangle \leftrightarrow |m\rangle$ and $|m\rangle \leftrightarrow |e\rangle$. The $|g\rangle \leftrightarrow |e\rangle$ transitions are forbidden.

1) Assuming that you can treat the system as two 2-level atoms with a shared level, show/explain in detail that in the dressed atom picture the Hamiltonian is given by (Ω is real, and $\delta = \omega_{RF} - \Delta E/\hbar$ is the detuning of the driving RF magnetic field)

$$H = \hbar \begin{bmatrix} 2\delta & \Omega/2 & 0 \\ \Omega/2 & \delta & \Omega/2 \\ 0 & \Omega/2 & 0 \end{bmatrix}$$

Also, specify the basis for the Hamiltonian.

2) Find the eigen-energies and eigen-states of the Hamiltonian for $\delta=0$.

3) Consider an initial state of the system where the atom is in the $|g\rangle$ state at t=0. Derive expressions for the probabilities to be in states $|g\rangle$, $|m\rangle$, and $|e\rangle$ as a function of time for $\delta=0$, and plot these probabilities as a function of time.

Extra Graduate Student Problem

Problem #4: Deriving the 2-Level Dressed Atom Hamiltonian

In this problem, you will derive the dressed atom Hamiltonian without resorting to dressed atom theory or its "atom + photon" basis, i.e. you will only use the $\{|g\rangle, |e\rangle\}$ basis (for the ground and excited states of a 2-level atom). You will work in the Schrodinger picture.

The standard 2-level atom Hamiltonian with energies E_g and E_e for the ground and excited atomic states is given by

$$H_{atom} = \begin{bmatrix} E_g & 0 \\ 0 & E_e \end{bmatrix}$$

The interaction Hamiltonian for the interaction of the atom with an oscillating electromagnetic field at frequency ω_l is given by (here Ω is the Rabi frequency for the interaction)

$$H_{int} = \hbar \begin{bmatrix} 0 & \Omega \\ \Omega^* & 0 \end{bmatrix} \cos \omega_l t$$

We will write the time dependence $c_g(t)$ and $c_e(t)$ of the $|g\rangle$ and $|e\rangle$ amplitudes, respectively, of the atomic wavefunction as (here $\omega_{g,e} = E_{g,e}/\hbar$)

$$|\psi(t)\rangle = c_g(t) e^{-i\omega_g t} |g\rangle + c_e(t) e^{-i\omega_e t} |e\rangle \tag{1}$$

(a) Write down the Schrodinger equation for $|\psi(t)\rangle$.

(b) Apply the Rotating Wave Approximation and show that you obtain the following equations for $c_g(t)$ and $c_e(t)$ (here $\delta = \omega_l - \omega_{eg}$, with $\omega_{eg} = \omega_e - \omega_g$):

$$i\hbar \frac{d}{dt} c_g(t) = c_e(t) \frac{\hbar\Omega}{2} e^{+i\delta t} \tag{2}$$

$$i\hbar \frac{d}{dt} c_e(t) = c_g(t) \frac{\hbar\Omega^*}{2} e^{-i\delta t} \tag{3}$$

(c) Next you will go to the "rotating frame" by introducing the rotating frame amplitudes $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$. The rotating frame transformation is defined as

$$\tilde{c}_g(t) = c_g(t)$$
$$\tilde{c}_e(t) = c_e(t) e^{+i\delta t}$$

Using the rotating frame amplitudes, show that equations (2) and (3) can be written in the form of a Schrodinger-like equation based on the dressed atom Hamiltonian:

$$i\hbar \frac{d}{dt} \begin{pmatrix} \tilde{c}_g(t) \\ \tilde{c}_e(t) \end{pmatrix} = \hbar \begin{bmatrix} 0 & \Omega/2 \\ \Omega^*/2 & -\delta \end{bmatrix} \begin{pmatrix} \tilde{c}_g(t) \\ \tilde{c}_e(t) \end{pmatrix}$$

(d) Write down the time dependence of the atomic wavefunction $|\psi(t)\rangle$ from equation 1 in terms of the rotating frame amplitudes $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$, i.e. instead of $c_g(t)$ and $c_e(t)$.

Note: The Bloch sphere picture and dynamics describe $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$.

Also $P_g = |\langle g|\psi(t)\rangle|^2 = |c_g(t)|^2 = |\tilde{c}_g(t)|^2$ & $P_e = |\langle e|\psi(t)\rangle|^2 = |c_e(t)|^2 = |\tilde{c}_e(t)|^2$.