

Tuesday, September 30, 2025

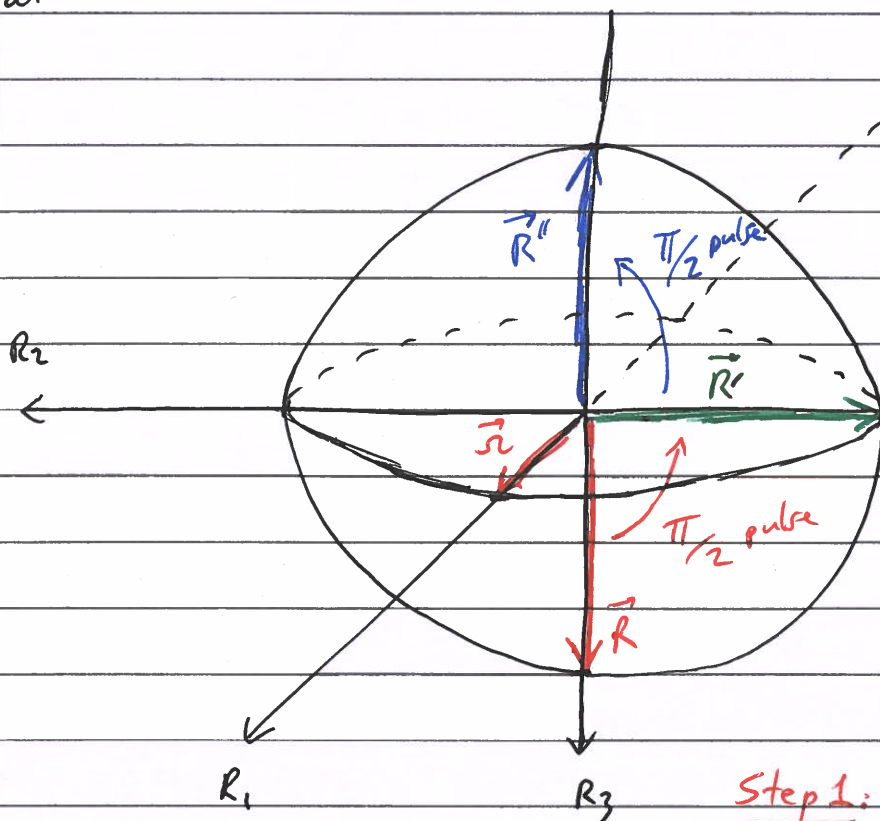
Bloch Sphere Example : Atomic clock / Ramsey Interferometer

Invented by Norman Ramsey in 1949 (Nobel 1989)
"method of separated oscillatory fields"

Suppose $\delta = 0$: $\vec{\Omega} = (\Omega, 0, \delta = 0)$

recall: $\frac{d\vec{R}}{dt} = \vec{\Omega} \times \vec{R}$

step 4: $|\psi''\rangle = |e\rangle$



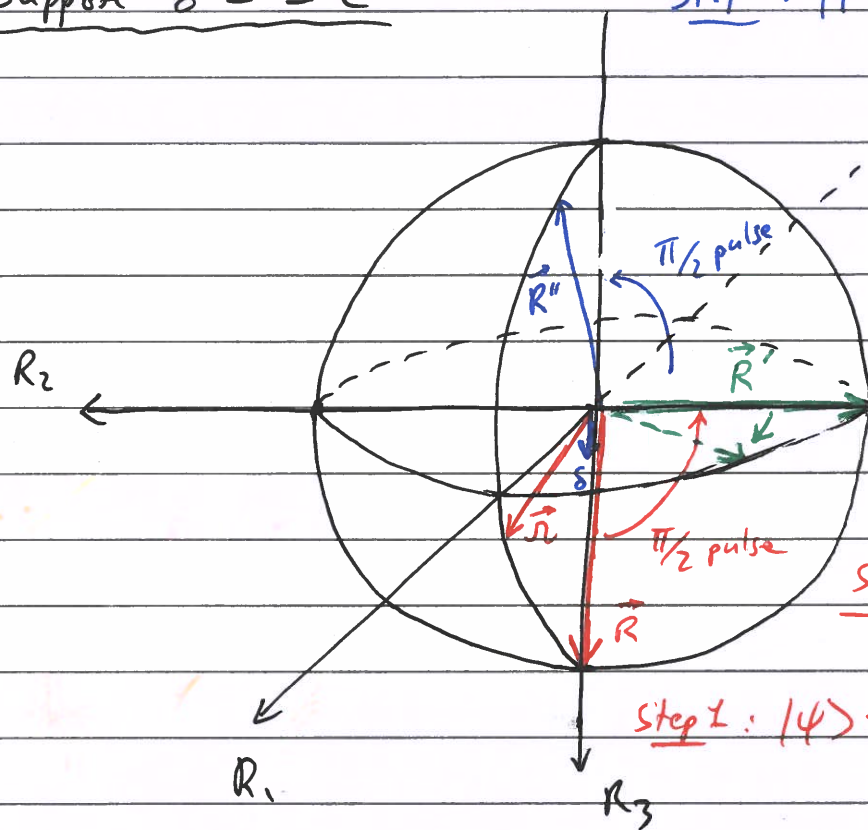
Step 3: wait time Δt
[Counted with the same oscillator used for $\pi/2$ pulse]

Step 2: $\pi/2$ pulse $\theta = \pi/2$
 $|\psi'\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$
 $= \frac{1}{\sqrt{2}}|g\rangle + \frac{e^{-i\theta}}{\sqrt{2}}|e\rangle$

Step 1: $|\psi\rangle = |g\rangle$

Suppose $\delta = \pm \epsilon$

step 4: $|\psi''\rangle = |e\rangle + \alpha(\delta t \delta) |g\rangle$



step 3: wait time δt with same oscillator used for $\pi/2$ pulses
 $\hookrightarrow \vec{R}'$ precesses at angular velocity δ .

key step

step 2: $|\psi\rangle \approx \frac{1}{\sqrt{2}} |g\rangle + \frac{e^{i\phi(\epsilon)}}{\sqrt{2}} |e\rangle$

step 1: $|\psi\rangle = |g\rangle$

$\Rightarrow$ By looking at the population ratio $\frac{|c_e|^2}{|c_g|^2}$ you can correct δ !!

Spontaneous Emission

I. Basic Physics

Recall the dressed atom Hamiltonian:

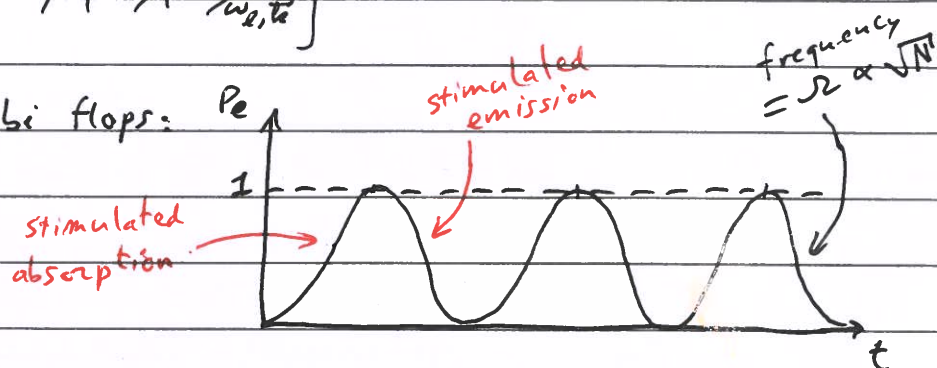
$$H = H_{\text{atom}} + H_{\text{EM-field}} + H_{\text{interaction}}$$

$$= \begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix} + \hbar \omega_{e,g} \begin{pmatrix} N+1 & 0 \\ 0 & N \end{pmatrix} + \hbar \begin{pmatrix} 0 & \Omega/2 \\ \Omega^*/2 & 0 \end{pmatrix}$$

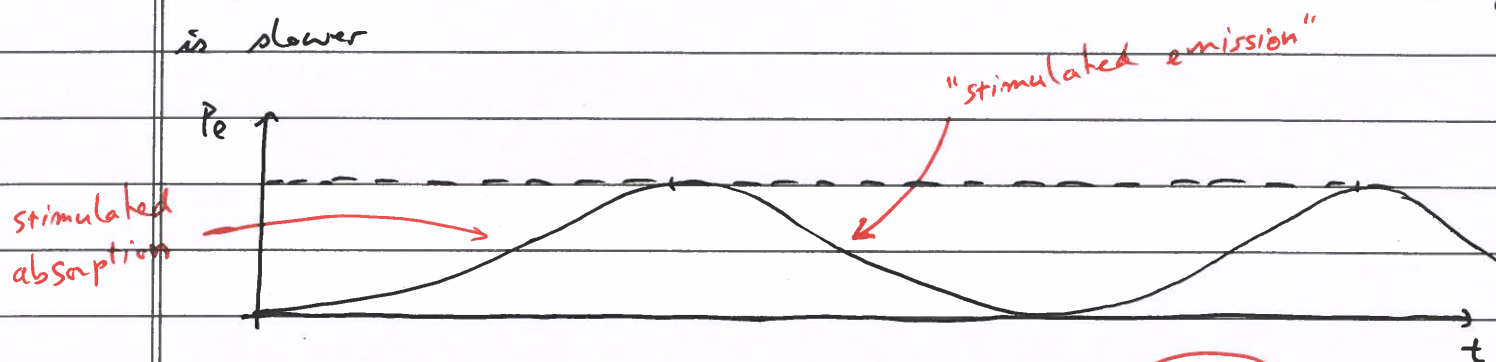
note: $\Omega \rightarrow \Omega(\sqrt{N}) \propto \sqrt{N}$

basis: $\{ |g\rangle |N+1\rangle_{\omega_{e,g}}, |e\rangle |N\rangle_{\omega_{e,g}} \}$

Atom-field system Rabi flops:



Suppose that we have a $N=1$ laser, then the Rabi-flopping is slower



The basis is $\{ |g\rangle |1\rangle_{\omega_{e,g}}, |e\rangle |0\rangle_{\omega_{e,g}} \}$

energetically, we can include $|e\rangle |0\rangle_{\omega_{e,g}}$ as well

photon vacuum state

So the basis is really

during stimulated emission
photon could go to $k_c = k_c'$

$$\left\{ |g\rangle|1\rangle_{\omega_c, k_c}, |e\rangle|0\rangle_{\omega_c, k_c}, |e\rangle|0\rangle_{\omega_c, k_c' \neq k_c}, \dots \right.$$

infinite number of $k_c' \neq k_c$

$$\dots |g\rangle|1\rangle_{\omega_c, k_c'} \}$$

(II) Spontaneous Emission

In other words, an excited atom can undergo "stimulated emission" by the photon vacuum in any direction.

- An atom in an excited state will tend to decay into a superposition of many $|g\rangle|1\rangle_{k_c}$ "one" photon states.
(all pointing in different directions).

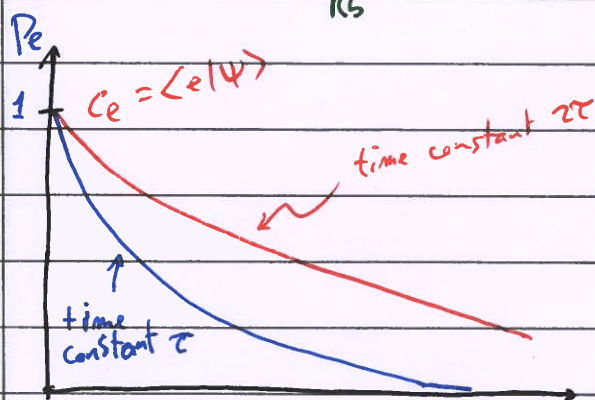
- The more states into which an atom can "Rabi flop" or leak into, the faster the decay.

↳ The decay is exponential with time constant τ .

For 2-level atom:

$$\tau = 1/\gamma = \frac{3\pi\epsilon_0\hbar c^3}{|\langle e|R|g\rangle|^2 \omega_{eg}^3}$$

$$\tau_{Rb} \approx 27 \text{ ns}$$



density of photon k_c' states scales like ω_{eg}^3

$$\frac{dP_e}{dt} = -\gamma P_e$$

$$\frac{dC_e}{dt} = -\frac{\gamma}{2} C_e$$

III) Derivation

[originally done by Wigner & Weisskopf]

[Method: Show that decay is exponential, i.e. $\frac{dC_e}{dt} = -\frac{\lambda}{2} C_e$]

Dressed atom wavefunction for decaying excited states:

$$| \psi(t) \rangle = c_e(t) | e \rangle + \sum_{\gamma \text{ photon states}} c_{g,\gamma}(t) e^{-i S_{\gamma} t} | g \rangle | 1 \rangle_{\gamma}$$

with $S_\delta = \omega_\delta - \omega_{\delta-1}$

We would like to write $\delta_r = 0$ to conserve energy, but since the excited state exists for a finite time, then there ~~is~~ is an energy spread.

The Schrodinger equation becomes:

Handwritten diagram illustrating a quantum system with two modes, showing energy levels and transitions. The diagram is divided into four vertical sections by dashed lines.

- Leftmost section:** Energy levels for mode 1, labeled $C_{g, \delta_1} e^{-i\delta_1 t}$, $C_{g, \delta_2} e^{-i\delta_2 t}$, ..., $C_{g, \delta_N} e^{-i\delta_N t}$, and C_e .
- Second section:** Energy levels for the combined system, labeled $S_{\delta_1/2}$, $S_{\delta_2/2}$, ..., $S_{\delta_N/2}$, and C_e .
- Third section:** Energy levels for the combined system, labeled $S_{\delta_1/2}$, $S_{\delta_2/2}$, ..., $S_{\delta_N/2}$, and C_e .
- Rightmost section:** Energy levels for mode 2, labeled $C_{g, \delta_1} e^{-i\delta_1 t}$, $C_{g, \delta_2} e^{-i\delta_2 t}$, ..., $C_{g, \delta_N} e^{-i\delta_N t}$, and C_e .

Transitions are indicated by dashed lines between the middle two sections and by solid lines between the left and right sections. The label "ith $\frac{d}{dt}$ " is on the far left.

$$i \frac{d}{dt} C_e(t) = \sum_{\gamma \text{ states}} C_{g,\gamma}(t) \frac{\Omega_\gamma}{2} e^{-i\delta_\gamma t} \quad (1)$$

$$i \frac{d}{dt} C_{g,\gamma}(t) \cdot e^{-i\delta_\gamma t} + \cancel{\delta_\gamma \cdot e^{-i\delta_\gamma t} C_{g,\gamma}(t)} = \cancel{\delta_\gamma C_{g,\gamma} e^{-i\delta_\gamma t}} + \frac{\Omega_\gamma}{2} C_e(t)$$

$$\hookrightarrow i \frac{d}{dt} C_{g,\gamma}(t) = \frac{\Omega_\gamma}{2} C_e(t) e^{i\delta_\gamma t} \quad (2)$$

Stopped Here

infinite number of equations
(1 for each γ photon state)

Next, integrate (2)

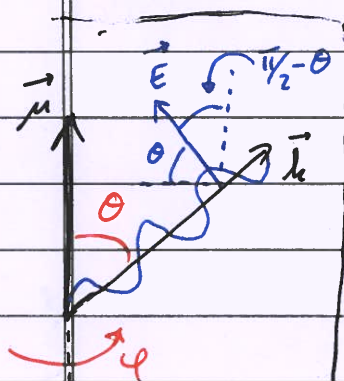
$$C_{g,\gamma}(t) = -i \frac{\Omega_\gamma}{2} \int_0^t e^{i\delta_\gamma t'} C_e(t') dt' \quad (3)$$

then substitute to (1):

$$\frac{d}{dt} C_e(t) = - \sum_{\gamma} \frac{|\Omega_\gamma|^2}{4} \int_0^t e^{-i\delta_\gamma(t-t')} C_e(t') dt'$$

plan: evaluate this part in the hope of pulling $C_e(t')$ term out of integral.

Recall: $\Omega_\gamma = \frac{\langle g | H_{int} | e \rangle}{\hbar} = \frac{\langle g | -e \vec{R} \cdot \vec{E}_\gamma | e \rangle}{\hbar}$



$$= -e \underbrace{\langle g | \vec{R} | e \rangle}_{\substack{\text{electric dipole} \\ \text{transition moment} \\ = \vec{\mu}_{eg}}} \cdot \frac{\vec{E}_\gamma}{\hbar} = -\vec{\mu}_{eg} \cdot \frac{\vec{E}_\gamma}{\hbar}$$

$$= -\frac{1}{\hbar} |\vec{\mu}_{eg}| |\vec{E}_\gamma| \sin \theta$$

θ gives the direction of $\vec{k}$ not $\vec{E}$.