Tuesday, September 30, 2025

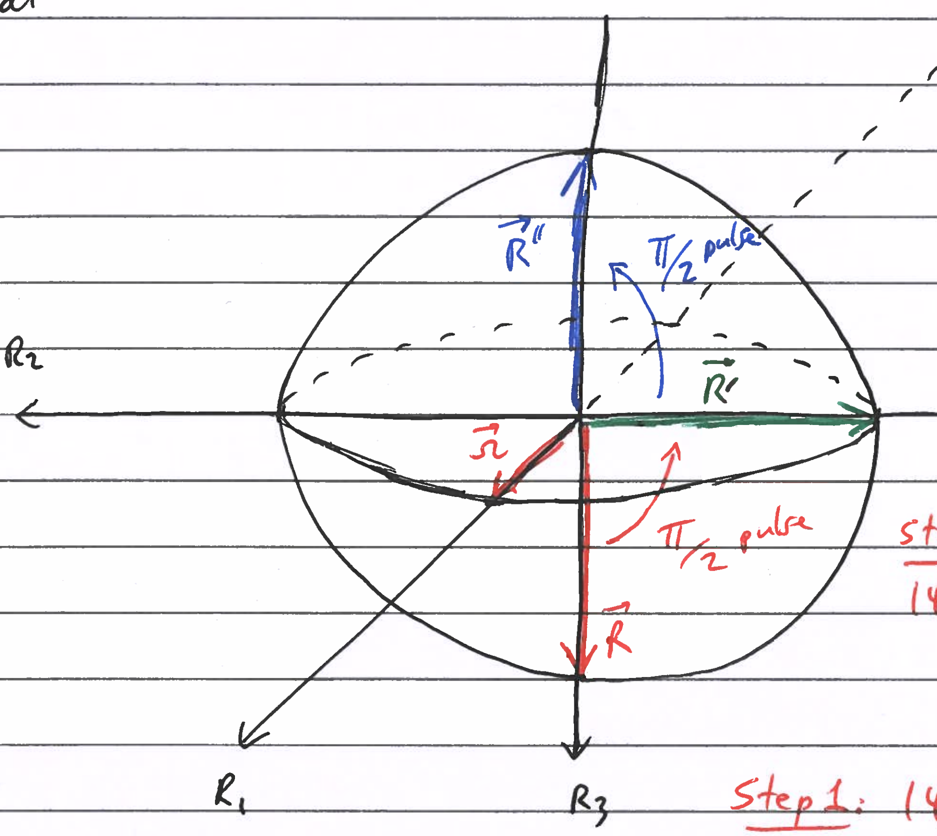
Bloch Sphere Example : Atomic clock / Ramsey Interferometer

Invented by Norman Ramsey in 1949 (Nobel 1989)
"method of separated oscillatory fields"

Suppose $\delta = 0$: $\vec{\Omega} = (\Omega, 0, \delta = 0)$

recall: $\dfrac{d\vec{R}}{dt} = \vec{\Omega} \times \vec{R}$

step 4: $|\psi''\rangle = |e\rangle$



Step 3: wait time Δt
[Counted with the same oscillator used for $\pi/2$ pulse]

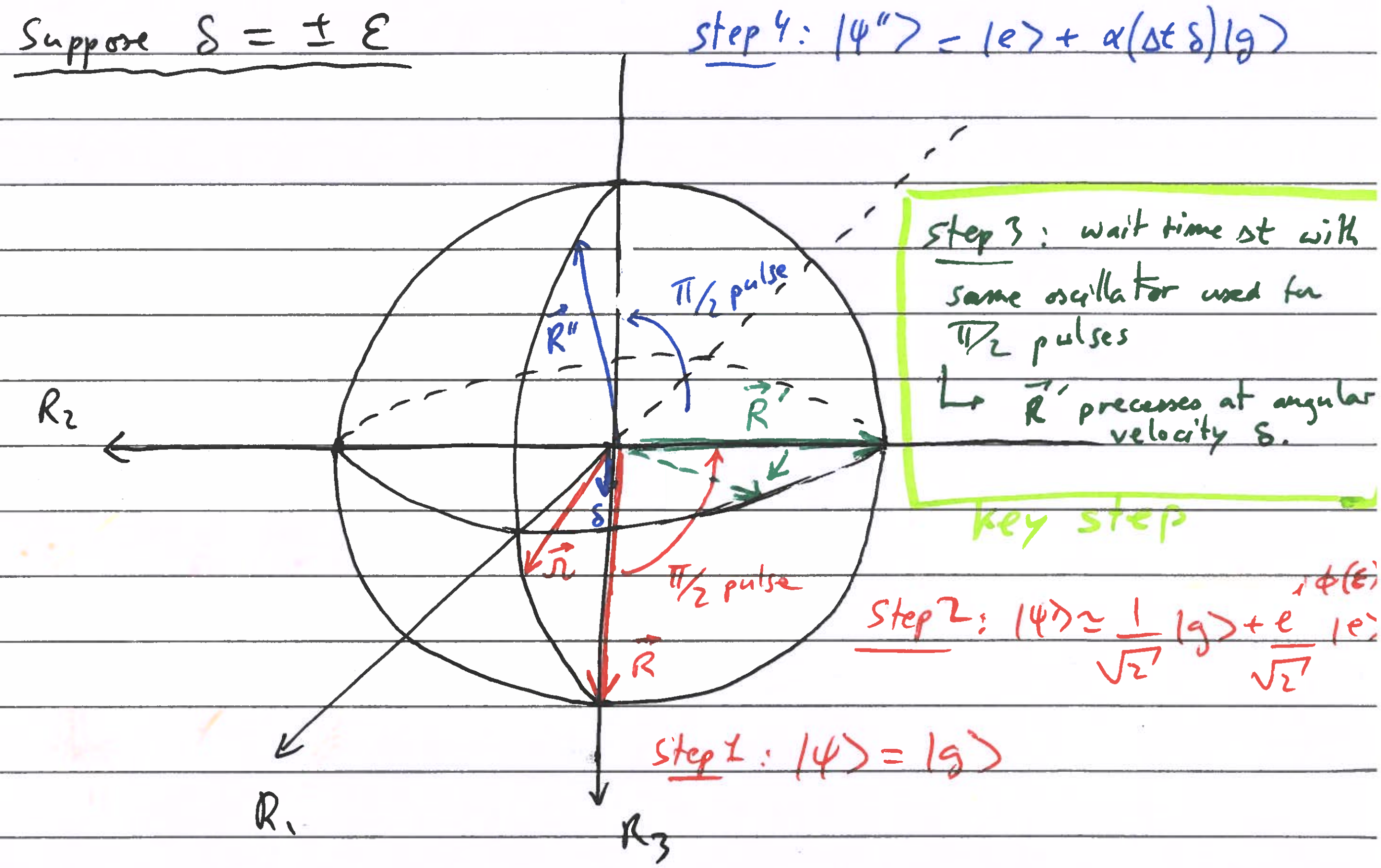
step 2: $\pi/2$ pulse $\theta = \pi/2$

$|\psi'\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$

$= \frac{1}{\sqrt{2}}|g\rangle + \frac{e^{-i\theta}}{\sqrt{2}}|e\rangle$

Step 1: $|\psi\rangle = |g\rangle$

Suppose $\delta = \pm \varepsilon$



$\Rightarrow$ By looking at the population ratio $\dfrac{|c_e'|^2}{|c_g'|^2}$ you can correct δ!!

Spontaneous Emission

I. Basic Physics

Recall the dressed atom Hamiltonian:

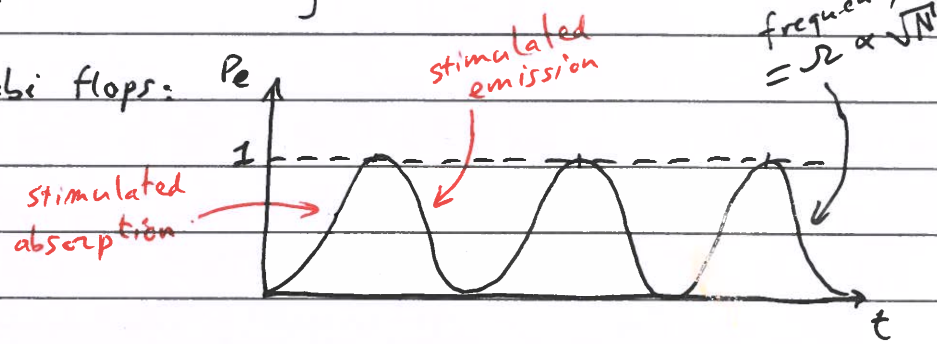
$$H = H_{atom} + H_{EM\text{-}field} + H_{interaction}$$

$$= \begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix} + \hbar\omega_{\ell,\vec{k}} \begin{pmatrix} N+1 & 0 \\ 0 & N \end{pmatrix} + \hbar \begin{pmatrix} 0 & \Omega/2 \\ \Omega^*/2 & 0 \end{pmatrix}$$

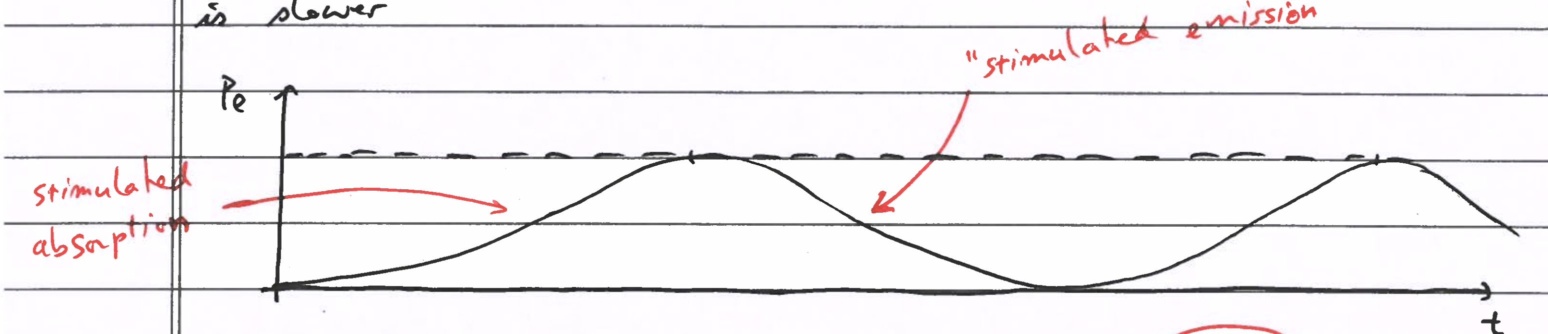
note: $\Omega \rightarrow \Omega(\sqrt{N}) \propto \sqrt{N}$

basis: $\left\{ |g\rangle|N+1\rangle_{\omega_\ell,\vec{k}} \, , \, |e\rangle|N\rangle_{\omega_\ell,\vec{k}} \right\}$

Atom-field system Rabi flops:



Suppose that we have a $N=1$ laser, then the Rabi-flopping is slower



The basis is $\left\{ |g\rangle|1\rangle_{\omega_\ell,\vec{k}} \, , \, \underbrace{|e\rangle|0\rangle_{\omega_\ell,\vec{k}}}_{\text{energetically, we can include } |e\rangle|0\rangle_{\omega_\ell,\vec{k}'} \text{ as well}} \right\}$

photon vacuum state

So the basis is really

$$\left\{ |g\rangle|1\rangle_{\omega_\ell,\vec{k}},\ |e\rangle|0\rangle_{\omega_\ell,\vec{k}},\ \underbrace{|e\rangle|0\rangle_{\omega_\ell,\vec{k}'\neq\vec{k}}}_{\text{infinite number of } \vec{k}'\neq\vec{k}},\ \dots \right.$$
$$\left. \dots\ |g\rangle|1\rangle_{\omega_\ell,\vec{k}'} \right\}$$

during "stimulated emission" photon could go to $\vec{k}=\vec{k}'$

(II) Spontaneous Emission

In other words, an excited atom can undergo "stimulated emission" by the photon vacuum in any direction.

- An atom in an excited state will tend to decay into a superposition of many $|g\rangle|1\rangle_{\vec{k}'}$ "one" photon states. (all pointing in different directions).

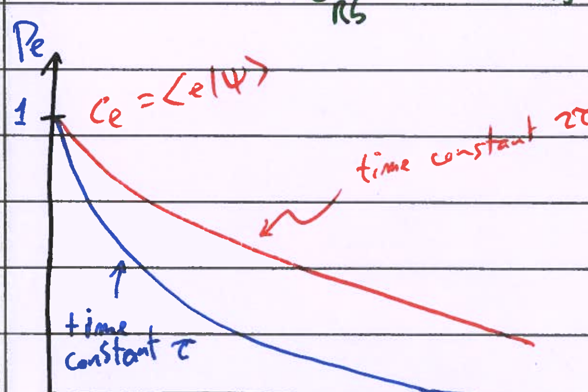
- The more states into which an atom can "Rabi flop" or leak into, the faster the decay.

$\rightarrow$ The decay is exponential with time constant τ.
For 2-level atom:

$$\tau = 1/\gamma = \frac{3\pi\varepsilon_0 \hbar c^3}{|\langle e|R|g\rangle|^2\, \omega_{eg}^3}$$

$\tau_{Rb} \simeq 27\,\text{ns}$

density of photon $\vec{k}'$ state scales like ω_{eg}^3



$$\frac{dP_e}{dt} = -\gamma P_e$$

$$\frac{dC_e}{dt} = -\frac{\gamma}{2} C_e$$

III Derivation [originally done by Wigner & Weisskopf]

[Method: Show that decay is exponential, i.e. $\frac{d}{dt} C_e = -\frac{\gamma}{2} C_e$]

Dressed atom wavefunction for decaying excited states:

$$|\psi(t)\rangle = C_e(t)|e\rangle + \sum_{\gamma \text{ photon states}} C_{g,\gamma}(t) \underbrace{e^{-i\delta_\gamma t}} |g\rangle|1\rangle_\gamma$$

We would like to write $\delta_\gamma = 0$ to conserve energy, but since the excited state exists for a finite time, then there is an energy spread.

with $\delta_\gamma = \omega_\gamma - \omega_{eg}$

The Schrodinger equation becomes:

$$i\hbar \frac{d}{dt} \begin{pmatrix} C_{g,\gamma_1} e^{-i\delta_1 t} \\ C_{g,\gamma_2} e^{-i\delta_2 t} \\ \vdots \\ C_{g,\gamma_N} e^{-i\delta_N t} \\ C_e \end{pmatrix} = \hbar \begin{pmatrix} \delta_1 & & & 0 & \Omega_{\gamma_1}/2 \\ & \delta_2 & & & \Omega_{\gamma_2}/2 \\ & & \ddots & & \vdots \\ 0 & & & \delta_N & \Omega_{\gamma_N}/2 \\ \frac{\Omega^*_{\gamma_1}}{2} & \frac{\Omega^*_{\gamma_2}}{2} & \cdots & \frac{\Omega^*_{\gamma_N}}{2} & 0 \end{pmatrix} \begin{pmatrix} C_{g,\gamma_1} e^{-i\delta_1 t} \\ C_{g,} e^{-i\delta_2 t} \\ \vdots \\ C_{g,\gamma_N} e^{-i\delta_N t} \\ C_e \end{pmatrix}$$

$$\begin{cases} i\dfrac{d}{dt}c_e(t) = \displaystyle\sum_{\gamma\,\text{states}} c_{g,\gamma}(t)\,\frac{\Omega_\gamma^*}{2}\,e^{-i\delta_\gamma t} & (1) \\[2ex] i\dfrac{d}{dt}c_{g,\gamma}(t)\cdot e^{-i\delta_\gamma t} + \cancel{\delta_\gamma\cdot e^{-i\delta_\gamma t}}\,c_{g,\gamma}(t) = \cancel{\delta_\gamma\, c_{g,\gamma}\, e^{-i\delta_\gamma t}} + \dfrac{\Omega_\gamma}{2}c_e(t) \end{cases}$$

$$\hookrightarrow\; i\frac{d}{dt}c_{g,\gamma}(t) = \frac{\Omega_\gamma}{2}\cdot c_e(t)\,e^{i\delta_\gamma t} \qquad (2)$$

infinite number of equations (1 for each γ photon state)

Stopped Here

Next, integrate (2)

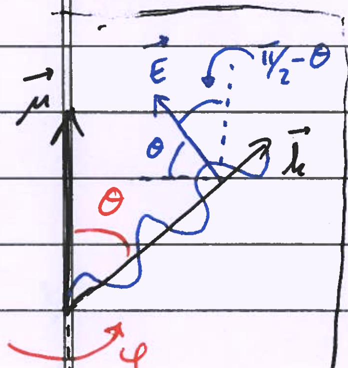
$$c_{g,\gamma}(t) = -i\,\frac{\Omega_\gamma}{2}\int_0^t e^{i\delta_\gamma t'}\,c_e(t')\,dt' \qquad (3)$$

then substitute to (1):

$$\frac{d}{dt}c_e(t) = -\sum_\gamma \underbrace{\frac{|\Omega_\gamma|^2}{4}\int_0^t e^{-i\delta_\gamma(t-t')}}\,c_e(t')\,dt'$$

Plan: evaluate this part in the hope of pulling $c_e(t')$ term out of integral.

Recall: $\Omega_\gamma = \dfrac{\langle g|H_{int}|e\rangle}{\hbar} = \dfrac{\langle g|-e\vec{R}\cdot E_\gamma|e\rangle}{\hbar}$



$$= \frac{\underbrace{-e\langle g|\vec{R}|e\rangle}_{\substack{\text{electric dipole} \\ \text{transition moment} \\ =\vec{\mu}_{eg}}}\cdot\vec{E}_\gamma}{\hbar} = -\vec{\mu}_{eg}\cdot\frac{\vec{E}_\gamma}{\hbar}$$

$$= -\frac{1}{\hbar}|\vec{\mu}_{eg}||\vec{E}_\gamma|\sin\theta$$

θ gives the direction of $\vec{k}$ not $\vec{E}$.