Tuesday, October 21, 2025

Monte Carlo Wavefunctions — Quantum Trajectories
(quantum jumps)

The "Schrodinger equation" for a 2-level dressed atom in a laser field with spontaneous emission is

$$i\hbar \frac{d}{dt}\begin{pmatrix} C_g \\ C_e \end{pmatrix} = \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega^*/2 & -\delta/2 \end{pmatrix}\begin{pmatrix} C_g \\ C_e \end{pmatrix} - \begin{pmatrix} 0 \\ i\hbar \frac{\gamma}{2} C_e \end{pmatrix}$$

$$= \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega^*/2 & -\delta/2 - i\gamma/2 \end{pmatrix}\begin{pmatrix} C_g \\ C_e \end{pmatrix}$$

effectively, $H = \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega^*/2 & -\frac{\delta}{2} - i\frac{\gamma}{2} \end{pmatrix}$

non-Hermitian

does not conserve probability

(complex eigenenergies)

time evolution: $e^{-i\left(\frac{iE_e}{\hbar}\right)t} = e^{\pm\frac{E_e t}{\hbar}}$

How do the states evolve in time?

Before a photon is spontaneously emitted:

$$|\Psi_0(t)\rangle = \left[C_g(t)\,|g\rangle_a |N+1\rangle_\ell + C_e(t)\,|e\rangle_a |N\rangle_\ell\right] \otimes |0\rangle_{\text{all other vacuum photon states}}$$

$C_g(t)$ & $C_e(t)$ are given by the previous "Schrodinger equation".

After a spontaneous photon is emitted at time $t = t_\gamma$, then

$$|\Psi_1(t=t_\gamma)\rangle = |g\rangle \otimes \Big[\underbrace{\sum_{k,\varepsilon}}_{\text{all possible "1" photon states}} \alpha_{k,\varepsilon}\, |k, \varepsilon, 1\rangle$$

photon polarization

momentum

accounts for dipole spatial emission distribution

Actually, the system (atom + laser + universe) could be in a superposition of these 2 states:
(leaked photon not yet detected)

$$|\Psi(t)\rangle = \underbrace{\sqrt{1-\beta^2}}\,\underbrace{|\Psi_0(t)\rangle}_{\text{no photon emitted}} + \beta\,\underbrace{|\Psi_1(t)\rangle}_{\text{1 photon leaks into an infinite superposition of photon momentum states}}$$

for short times

Loss of probability from H ends up here

$$\beta = \underbrace{\sqrt{\gamma\,|C_e|^2\,dt}}_{\text{for short times}}$$

Environment Detector

However, we are not dealing with a closed system, but an open quantum system. The "leaked photon" could be detected at any moment by the environment [environment is like a giant detector].

Let's assume that the "environment detector" is 100% efficient and makes a measurement in a time dt immediately after a photon leaks out $\longrightarrow$ wavefunction collapses.

"photon not emitted state"

keeps evolving as before, but needs to be renormalized, since system is definitely here.

Immediately after collapse:

$$|\psi(t+dt)\rangle = \begin{cases} \psi_0(t+dt) \text{ with probability } p = 1-\beta^2 \\ \text{or} \\ \psi_1(t+dt) = |g\rangle\underbrace{|k,\varepsilon,1\rangle} \text{ with probability } p = \beta^2 \end{cases}$$

photon collapses to some k, ε state, but we don't pay attention to it.

$\hookrightarrow$ process repeats $\longrightarrow$ propagate a time dt again.

This story suggests the following algorithm for calculating the time evolution of the "atom + laser + universe/environment detector" system .... though we only pay attention to the atomic state.

Step 1: At time $= t$, the state of the system is

$$|\psi(t)\rangle = |\psi_0(t)\rangle$$
$$= c_g(t)|g\rangle + c_e(t)|e\rangle$$

Step 2: at time $= t+dt$, we have 2 possible trajectories for the wavefunction:

re-normalization

$$|\psi(t+dt)\rangle = \begin{cases} \text{case A}: \dfrac{1}{\sqrt{1-dp}}\left[c_g(t+dt)|g\rangle + c_e(t+dt)|e\rangle\right] & \text{with probability } 1-dp = 1-\beta^2 \\ \text{OR} \\ \text{Case B}: \quad |g\rangle \quad \text{with probability } dp \\ \text{with } dp = \gamma|c_e|^2 dt = \beta^2 \end{cases}$$

Q: How do you pick between case A & case B ?

A: "flip a coin": choose a random number $\varepsilon \in [0,1]$

$$\begin{cases} \text{if } \varepsilon > dp \Rightarrow |\psi(t+dt)\rangle = |\text{Case A}\rangle \\ \text{if } \varepsilon < dp \Rightarrow |\psi(t+dt)\rangle = |\text{case B}\rangle = |g\rangle \end{cases}$$

Condition on dt : $dt = \Delta t \ll \dfrac{1}{\Omega}, \dfrac{1}{\gamma}, \dfrac{1}{\delta}$

not infinitesimal (though that would be ideal)

Random number generator must work well for very small numbers, i.e. $\varepsilon \simeq 0$ ($< dp$) !!!