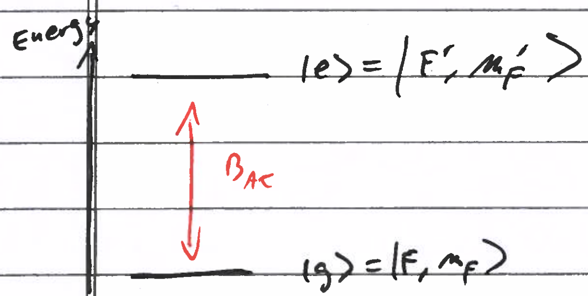
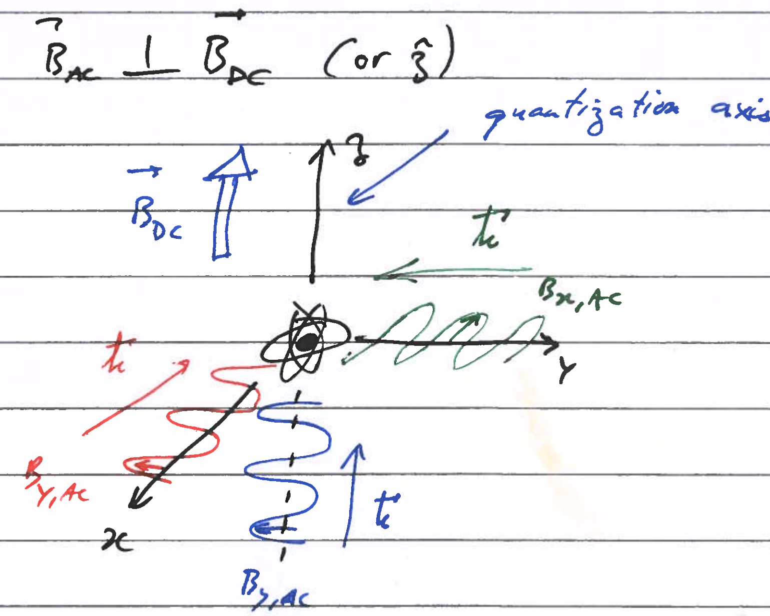
Thursday, October 30, 2025

mostly for RF and microwave transitions

Transition Selection Rules : M1 transitions (continued)

Case 2: σ- polarization $\vec{B}_{AC} \perp \vec{B}_{DC}$ (or $\hat{z}$)





Direction of polarization is important

Direction of propagation $\vec{k}$ is unimportant.

Start with $\begin{cases} F = F' \\ m_F \neq m_F' \end{cases}$

Rabi frequency: $$\Omega = \frac{\langle F, m_F' | H_{Zeeman,AC} | F, m_F \rangle}{\hbar}$$

$$= \langle F, m_F' | \frac{\mu_B}{\hbar^2} g_F \vec{F} \cdot \vec{B}_{AC} | F, m_F \rangle$$

$$= \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | F_x B_{x,AC} + F_y B_{y,AC} | F, m_F \rangle$$

But $F_x = (F_+ + F_-)/2$ and $F_y = (F_+ - F_-)/2i$

so $$\Omega = \frac{\mu_B g_F}{\hbar^2} \langle F, m_F' | \left(\frac{F_+ + F_-}{2}\right) B_x + \left(\frac{F_+ - F_-}{2i}\right) B_y | F, m_F \rangle$$

$$\Rightarrow \Omega = \frac{\mu_B g_F}{\hbar^2} \langle F, m_F' | F_+ \underbrace{\left(\frac{B_x - iB_y}{2}\right)}_{B_-} + F_- \underbrace{\left(\frac{B_x + iB_y}{2}\right)}_{B_+} | F, m_F \rangle$$

$$\Leftrightarrow \Omega = \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | F_+ B_- + F_- B_+ | F, m_F \rangle$$

where
$$\begin{cases} B_{+,AC} = \dfrac{B_x + iB_y}{2} = \sigma^+ \text{ polarization} = \text{right circularly polarized} \\ B_{-,AC} = \dfrac{B_x - iB_y}{2} = \sigma^- \text{ polarization} = \text{left circularly polarized} \end{cases}$$

⚠ Note: Definition does not depend on whether wave is going in $+\hat{k}$ or $-\hat{k}$ direction.

Recall that
$$\begin{cases} F_+ |F, m_F\rangle = \hbar\sqrt{F(F+1) - m_F(m_F+1)}\, |F, m_F+1\rangle \\ F_- |F, m_F\rangle = \hbar\sqrt{F(F+1) - m_F(m_F-1)}\, |F, m_F-1\rangle \end{cases}$$

Thus $\Omega \neq 0$ only if $\begin{cases} \Delta m_F = m_F - m_{F'} = +1 & \text{for } B_+ \\ \Delta m_F = m_F - m_{F'} = -1 & \text{for } B_- \end{cases}$

If $F \neq F'$, then $(L=0)$

$$\Omega = \frac{\mu_B g_s}{\hbar^2} \langle F', m_F' | S_+ B_{-AC} + S_- B_{+AC} | F, m_F\rangle$$

$\hookrightarrow$ then one can have $\Delta F = 0, \pm 1$

Summary for M1 transition:

$$\boxed{\text{For } \pi\text{-polarization} \longrightarrow \Delta F = \pm 1, \ \Delta m_F = 0}$$

$$\boxed{\text{For } \sigma\text{-polarization} \longrightarrow \Delta F = 0, \pm 1, \ \Delta m_F = \pm 1}$$

Selection Rules for E1 transitions

(most optical transitions)

[$\vec{B}_{DC} = B_{DC}\hat{z}$ still provides the quantization axis]

$$\Omega = \frac{-e}{\hbar}\underbrace{\langle n', L', J', F', m_F'|}_{\langle e|}\vec{E}_{ac}\cdot\vec{R}\underbrace{|n, L, J, F, m_F\rangle}_{|g\rangle}$$

For π-polarization $\iff \vec{E}_{AC} \parallel \vec{B}_{DC} \parallel \hat{z}$

$\Omega \neq 0$ for $\begin{cases}\Delta F = 0, \pm 1, \Delta m_F = 0 \\ \Delta L = \pm 1\end{cases}$

Exception: $\Omega = 0$ for $\begin{cases}\Delta F = 0, m_F = 0 \\ \Delta L = \pm 1\end{cases}$

For σ-polarization

$\Omega \neq 0$ for $\begin{cases}\Delta F = 0, \pm 1, \Delta m_F = \pm 1 \\ \Delta L = \pm 1\end{cases}$

note: $\sigma_+ : \Delta m_F = +1$
$\sigma_- : \Delta m_F = -1$

Cycling Transition (D_2 line): see overhead slides

Optical Pumping

$F=2 \to F'=3$ with $\sigma^\pm$ light $\Rightarrow$ all population goes to $m_F = +F$ or $m_F = -F$

$F=2 \to F'=2$ with $\sigma^\pm$ light $\to$ perfect alignment
↳ all population goes to $m_F = +F$ or $m_F = -F$
↳ transition goes "dark"

$F=2 \to F'=2$ with π light $\to$ perfect alignment
↳ all population goes to $m_F = 0$
↳ transition goes "dark"