PHYS 610: Electricity & Magnetism I
Due date: Thursday, March 31, 2016

Problem set #6

1. Variation on Stokes theorem

Prove the following integral theorem: $\int_S \hat{n} \times \vec{\nabla} f \, dS = \int_C f \, \overrightarrow{dl}$

Where f is a scalar function, S is surface with contour C, $\hat{n}$ is a unit vector locally perpendicular to S, and $\overrightarrow{dl}$ is a differential line element along C.

2. Green's identities

a) Use the divergence theorem to prove *Green's first identity*:

$$\int_V [\phi \vec{\nabla}^2 \psi + \vec{\nabla}\phi \cdot \vec{\nabla}\psi] \, d^3r = \int_S \phi \vec{\nabla}\psi \cdot \overrightarrow{dS}$$

$\phi(\vec{r})$ and $\psi(\vec{r})$ are arbitrary (well-behaved) scalar functions, and *V* is a volume with surface *S*.

b) Prove *Green's second identity*:

$$\int_V [\phi \vec{\nabla}^2 \psi - \psi \vec{\nabla}^2 \phi] \, d^3r = \int_S [\phi \vec{\nabla}\psi - \psi \vec{\nabla}\phi] \cdot \overrightarrow{dS}$$

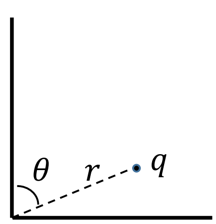
3. Mean value theorem for electrostatics

Consider a function $f(\vec{r})$ that obeys Laplace's equation $\vec{\nabla}^2 f = 0$. Show that $f(\vec{r})$ obeys the following average rule: The value of $f(\vec{r})$ at any point $\vec{r}$ is equal to the average of $f(\vec{r})$ over the surface of any sphere centered on $\vec{r}$.

Note: this result shows that $f(\vec{r})$ can have no local maximum or minimum, only saddle points at most.

4. Point charge near a corner

Two semi-infinite and grounded conducting planes meet at a right angle as seen edge-on in the sketch below. Find the charge induced on each plane when a point charge *q* is placed at a distance *r* from the corner with angle alpha as shown in the sketch.



5. Conducting sphere and a point charge

a) In class, we considered a point charge q at a distance s from a grounded conducting sphere of radius R ($R<s$). Now, consider the case of a conducting sphere which is held at a constant potential V_0 (in the presence of a nearby point charge). Show that you can model this situation by adding a second image charge, and use it to obtain a formula for the potential $V(\vec{r})$ everywhere outside of the sphere. Calculate the surface charge density and the total charge of the sphere.

b) Next we consider the case of an isolated (i.e. not explicitly maintained at a fixed potential) neutral conducting sphere in the presence of a point charge. Calculate the potential $V(\vec{r})$ produced by this arrangement (everywhere outside of the sphere). Calculate the surface charge density and verify that the total charge on the sphere is zero.

Calculate the force between the charge and the sphere. Is it attractive or repulsive?

Calculate the work (in Joules) to move a 1 C point charge from a distance of 1 m from a neutral conducting sphere of diameter 1 m to infinity.

4. More images charges

a) Compute the electrostatic potential at the location of a point charge q due to the surface charge induced by the point charge on two grounded parallel plane conductors separated by a distance d, assuming that the point charge lies midway between the two planes.

b) What is the electric potential energy of this arrangement?