PHYS 610: Electricity & Magnetism I
Due date: Thursday, February 15, 2018

Problem set #4

1. Adding a divergence term to the Lagrangian density
Consider the following modification of the Lagrange density $\mathcal{L}(\phi, \partial^\mu \phi)$,

$$\mathcal{L} \to \mathcal{L} + \partial_\mu G^\mu(\phi)$$

where G^μ is any vector function of the field $\phi(x^\mu)$.
a) Show that the Euler-Lagrange equations of motion are unchanged.
b) Show that this extra term leaves variations of the action integral unchanged.

2. Lagrangian density practice
Find the Euler-Lagrange equations of motion for the scalar field $\phi(x^\mu)$ if the Lagrangian density is $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$.

3. Jackson problem 12.14

4. A simple classical mechanics field theory in 1D
It is often possible to derive a field theory as the limit of a discrete system. Consider an infinite system of identical point masses *m*, separated by identical springs with spring constant *k* and equilibrium length a. Let η_i be the displacement from equilibrium of the ith point mass.
a) Derive the exact (particle) Lagrangian and Euler-Lagrange equations of motion for this classical system (non-relativistic).

b) Next consider the limit $m, a \to 0$, $k \longrightarrow \infty$, but with $\mu = m/a$ and $Y = ka$ held fixed. Now, replace η_i with a smooth function $\eta(x,t)$ and show that in this limit the Lagrangian may be written as the integral of a density

$$L = \int dx \frac{1}{2}\left[\mu\left(\frac{\partial \eta}{\partial t}\right)^2 - Y\left(\frac{\partial \eta}{\partial x}\right)^2\right]$$

and write down the corresponding Euler-Lagrange equation for the $\eta(x,t)$ field.

5. Particle tracking
A relativistic electron enters a strong uniform magnetic field (directed along the z-axis) at some angle θ. No electric field is present.
a) Derive an equation for the trajectory of the particle in time (in the lab frame of the magnet) from the Lorentz invariant form of the Lorentz force.
b) What are the numerical parameters of the trajectory for a 12.0 GeV electron in a 1.00 Tesla magnetic field if $\theta = \pi/2$.