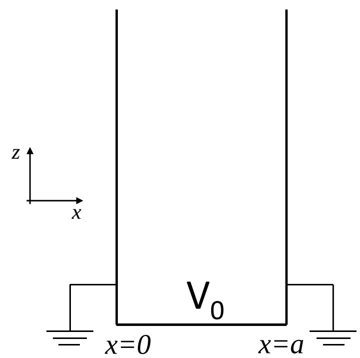
PHYS 610: Electricity & Magnetism I
Due date: Thursday, April 5, 2018

Problem set #8

1. Green's function inside a conducting spherical shell
Find the Green's function for calculating the electric potential $V(\vec{r})$ due to an arbitrary charge distribution inside a conducting spherical shell of radius R.

2. Conducting slot
Consider an infinitely long and deep slot formed by two grounded conductor plates at $x=0$ and $x=a$, as shown in the figure below. A third conductor plate, at $z=0$, is held at a constant potential V_0.



a) Find the potential inside the slot as a series solution.

b) Sum the series (hint: see Jackson section 2.10).

c) Determine the asymptotic behavior of the potential when $z>>a$.

3. Green's reciprocity theorem

a) Consider a charge distribution $\rho_1(\vec{r})$ that produces a potential $V_1(\vec{r})$, and a separate charge distribution $\rho_2(\vec{r})$ that produces a potential $V_2(\vec{r})$. The charge distributions are entirely unrelated, and are not even present at the same time, i.e. these two electrostatic situations are different "problems" and are not present simultaneously. Prove Green's reciprocity theorem:

$$\int_{\substack{all\\space}} \rho_1 V_2 \, d^3r = \int_{\substack{all\\space}} \rho_2 V_1 \, d^3r$$

Note: you may find it useful to calculate $\int \vec{E}_1 \cdot \vec{E}_2 \;\, d^3r$ in two different ways.

b) Consider two spatially separated and distinct conductors, A and B. If you charge up conductor A with charge Q (B remains uncharged), then the resulting potential of conductor B is V_{AB}. Alternatively, if instead you charge up conductor B with charge Q (A remains uncharged), then the resulting potential of conductor A is V_{BA}. Use Green's reciprocity theorem to show that $V_{AB} = V_{BA}$.

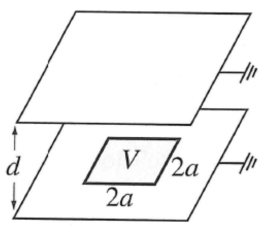
Note: this result makes no assumptions about the position or shapes of conductors A and B.

c) Both plates of a very large parallel plate capacitor are grounded and separated by a distance d. A point charge q is placed between them at a distance x from plate 1. Use Green's reciprocity theorem to calculate the induced charge on each plate.

Hint: For the charge on plate 1, use the actual situation, while for the charge on plate 2, remove the q, and set one of the conductors at potential V_0.

4.

A Potential Patch by Separation of Variables The square region defined by $-a \le x \le a$ and $-a \le y \le a$ in the $z = 0$ plane is a conductor held at potential $\varphi = V$. The rest of the $z = 0$ plane is a conductor held at potential $\varphi = 0$. The plane $z = d$ is also a conductor held at zero potential.



(a) Find the potential for $0 \le z \le d$ in the form of a Fourier integral.

(b) Find the total charge induced on the upper surface of the lower ($z = 0$) plate. The answer is very simple. Do not leave it in the form of an unevaluated integral or infinite series.

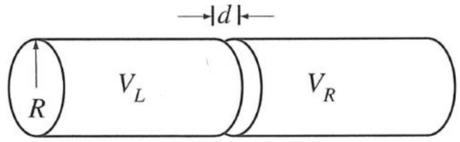
(c) Sketch field lines of $\mathbf{E}(\mathbf{r})$ between the plates.

5.

The Two-Cylinder Electron Lens Two semi-infinite, hollow cylinders of radius R are coaxial with the z-axis. Apart from an insulating ring of thickness $d \to 0$, the two cylinders abut one another at $z = 0$ and are held at potentials V_L and V_R. Find the potential everywhere inside both cylinders. You will need the integrals

$$\lambda \int_0^1 ds\, s\, J_0(\lambda s) = J_1(\lambda) \qquad \text{and} \qquad 2\int_0^1 ds\, s\, J_0(x_n s) J_0(x_m s) = J_1^2(x_n)\delta_{nm}.$$

The real numbers x_m satisfy $J_0(x_m) = 0$.



Plot the potential as a function r and z.

Note: Zangwill provides an alternate approach to this problem in "application 7.4".