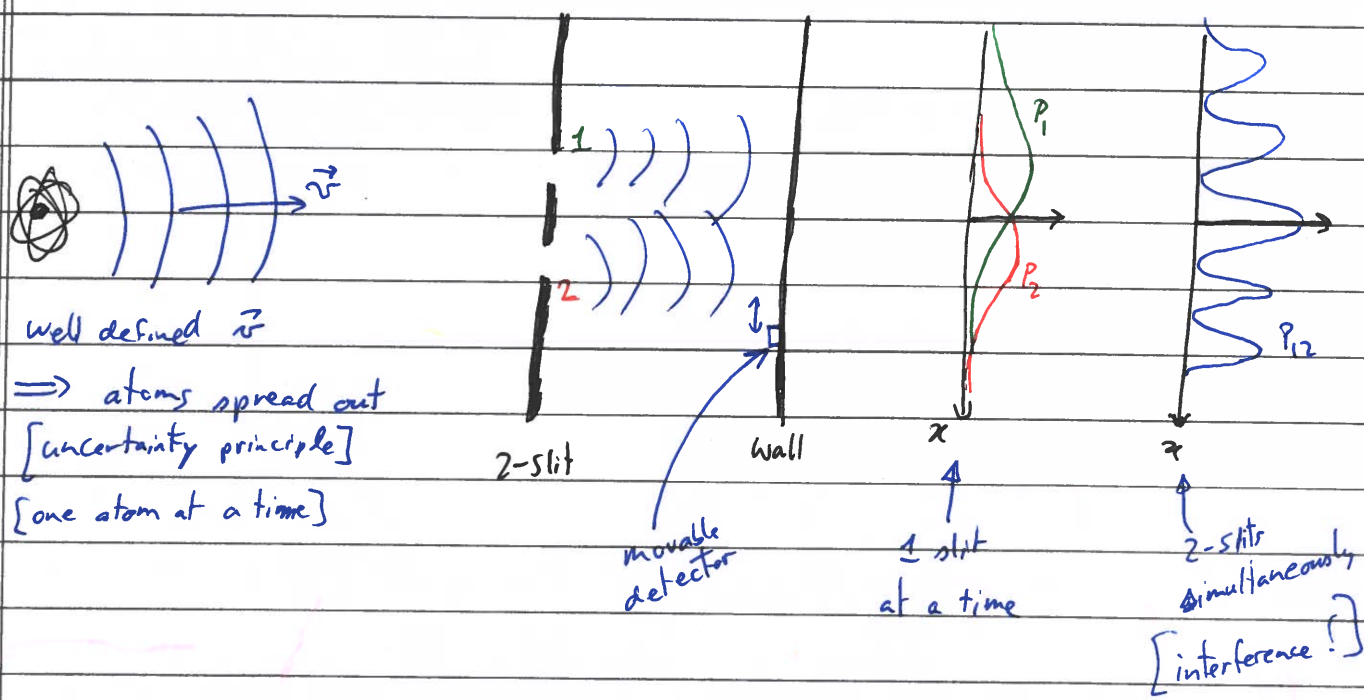
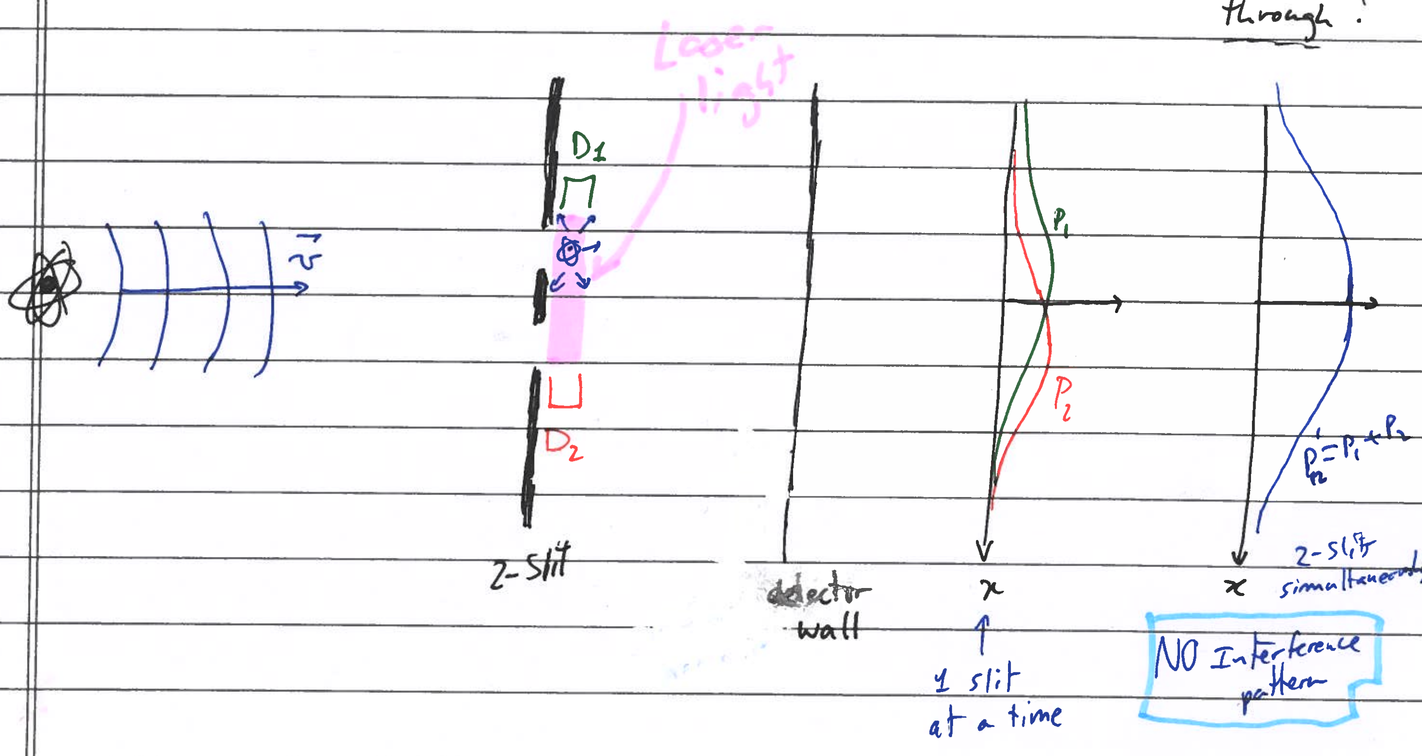
Thursday, August 27, 2026

Single particle interference + which way measurements

We direct single atoms (or electrons) with a very well defined velocity / kinetic energy at a double slit:



Q: What happens if we try to measure which slit the atom goes through?



$\Rightarrow$ Detecting/measuring which way the atom went destroys the the interference pattern.

$$P'_{12} = P_1 + P_2$$

A rudimentary "starter Quantum theory":

1) The probability of an event/detection is given by the modulus squared of a complex number called the probability amplitude:

$$P = |\varphi|^2 \geq 0 \quad [P \in \mathbb{R}]$$

2) When there are multiple ways/paths to generate an event, with no way to detect which path was taken, then the total probability ampl. is the sum of the probability amplitudes for ~~f~~ each path:

$$\varphi = \varphi_1 + \varphi_2$$

$$P = |\underbrace{\varphi_1 + \varphi_2}_{\varphi_1 \,\&\, \varphi_2 \text{ can add } \underline{\text{or}} \text{ subtract}}|^2 \qquad \text{note: } 0 \leq P \leq P_1 + P_2$$

3) However if there is a way of detecting which path was taken, then the ~~t~~ total probability is the sum of the probabilities of the individual ~~p~~ distinguishable paths:

$$P = P_1 + P_2 = \underbrace{|\varphi_1|^2 + |\varphi_2|^2}_{\text{subtraction not possible}}$$

note: $P \geq P_1, P_2$

Not covered

<u>Improved notation:</u>

Amplitude for an atom to start at the source $\underline{s}$ and arrive at position $\underline{x}$ on the detector wall:

$$\varphi = \langle x | s \rangle$$

The two paths for the atom to get to x give two different amplitudes:

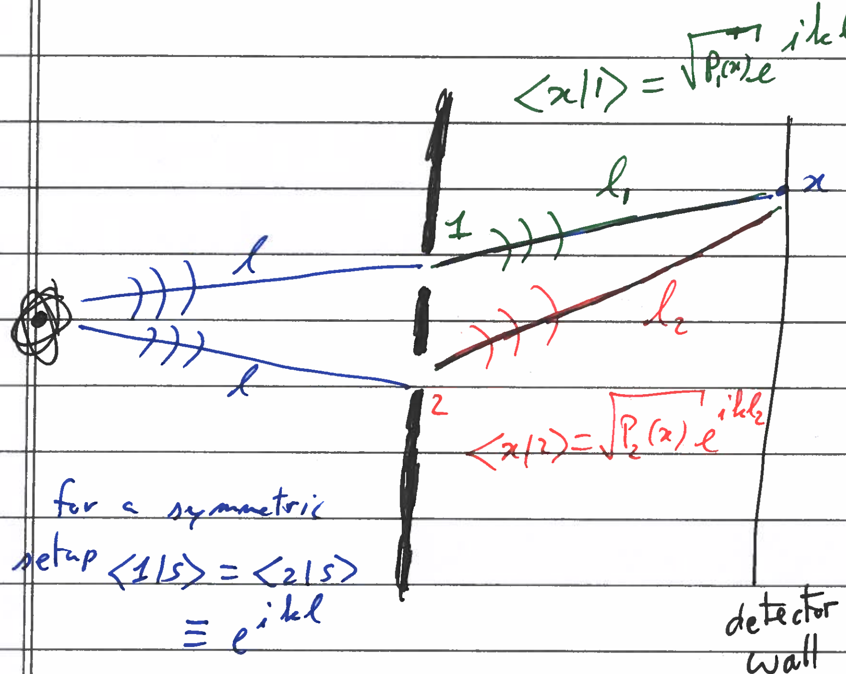
1) $\underbrace{s \longrightarrow \text{slit } 1}_{\langle 1|s\rangle} \underbrace{\longrightarrow x}_{\langle x|1\rangle}$ $\Big\}$ $\varphi_1 = \langle x|1\rangle\langle 1|s\rangle$

2) $\underbrace{s \longrightarrow \text{slit } 2}_{\langle 2|s\rangle} \underbrace{\longrightarrow x}_{\langle x|2\rangle}$ $\Big\}$ $\varphi_2 = \langle x|2\rangle\langle 2|s\rangle$

$$\Rightarrow \quad \varphi = \varphi_1 + \varphi_2$$

$$\Leftrightarrow \quad \boxed{\langle x|s\rangle = \langle x|1\rangle\langle 1|s\rangle + \langle x|2\rangle\langle 2|s\rangle}$$

Superposition of Paths 1 & 2.



for a symmetric setup $\langle 1|s\rangle = \langle 2|s\rangle \equiv e^{ikl}$

k = wavevector motivated by EM physics

$$\varphi = \langle x|s\rangle$$

$$= \underbrace{\sqrt{P_1(x)}}_{\text{envelope}}\, e^{ikl_1}\, e^{ikl}$$

$$+ \underbrace{\sqrt{P_2(x)}}_{\text{envelope (single slit diffraction)}}\, e^{ikl_2}\, e^{ikl}$$

not covered

$$P = |\psi|^2 = \Big|\underbrace{\sqrt{P_1(x)}}_{\simeq P_{12}(x)}\, e^{ikl_1} e^{ikl} + \underbrace{\sqrt{P_2(x)}}_{\simeq P_{12}(x)}\, e^{ikl_2} e^{ikl}\Big|^2$$

$$\simeq \underbrace{|e^{ikl}|^2}_{=1} \underbrace{\left|\sqrt{P_{12}(x)}\right|^2}_{P_{12}(x)} \left|e^{ikl_1} + e^{ikl_2}\right|^2$$

$$\simeq P_{12}(x) \underbrace{\left|e^{ik\frac{l_1}{2}}\right|^2}_{=1} \underbrace{\left|e^{ik\frac{l_2}{2}}\right|^2}_{=1} \Big|\underbrace{e^{ik\frac{l_1-l_2}{2}} + e^{ik\frac{l_2-l_1}{2}}}_{2\cos[k(l_1-l_2)]}\Big|^2$$

$$\simeq 4P_{12}(x) \cos^2[k(l_1-l_2)]$$

$$\Rightarrow \boxed{P \simeq 4 P_{12}(x) \cos^2[k\underbrace{(l_1-l_2)}_{\text{depends on } \underline{x}}]}$$

covered

two key concepts: 1) Superposition of two states at once
probability amplitudes

2) Measurement/detection destroys superposition.

Quiz on Tuesday
covering Sakurai section 1.1
p 1-10
"The Stern-Gerlach Experiment"