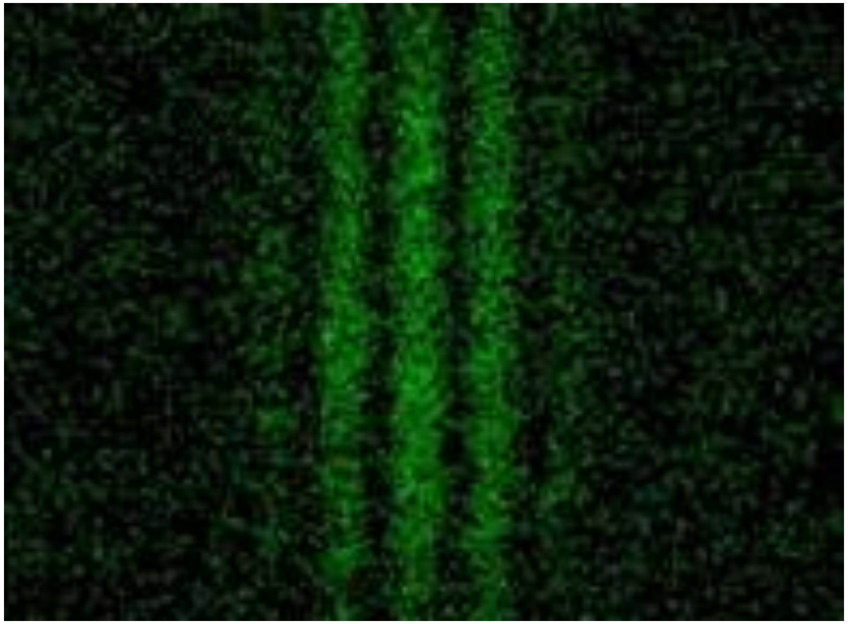
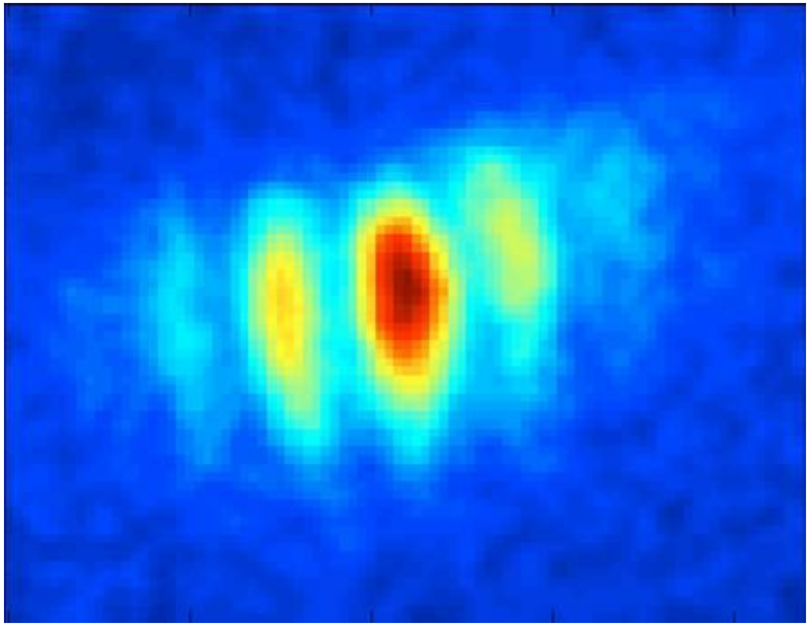
Physics 621

Graduate Quantum Mechanics I



[A. L. Weiss and T. L. Dimitrova, Swiss Physics Society, 2009.]



[Thywissen Lab, U. of Toronto, 2006.]

Instructor

Prof. Seth Aubin

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web: http://www.physics.wm.edu/~saubin/index.html

Office hours: Wednesdays 4-5 pm

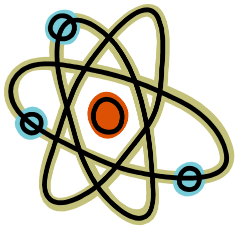
Russell Kamback

Office: room 069, Small Hall
e-mail: rmkamback@wm.edu

Office hours: Wednesdays noon – 1 pm

Course Objectives (I)

Introduce the **physical** and **mathematical** **foundations** of **quantum mechanics** and to calculate the behavior of particles/waves.



Topics:

- **Postulates** of quantum mechanics.
- Canonical **quantization** procedure.
- **Wavefunctions**, Hilbert spaces, **Dirac notation** (bras & kets).
- Schrodinger and **Heisenberg** representations.
- **Commutators** and uncertainty relations.
- Hamiltonian formalism, **Schrodinger's equation**.

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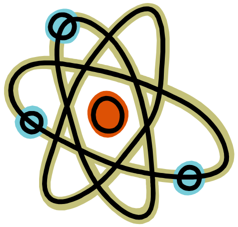

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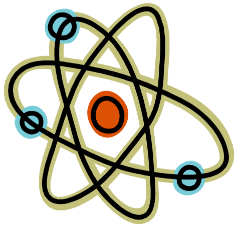
- Schrodinger and **Heisenberg** representations.
- **Commutators** and uncertainty relations.
- Hamiltonian formalism, **Schrodinger's equation**.
- **2-level systems**: avoided crossings, time evolution.
- **Generators** of translations, rotations, and time evolution.
- Theory of **angular momentum**, **spin**.
- **Addition** of angular momenta, Clebsch-Gordan coefficients.
- **Tentative:** Identical particles, bosons, fermions.

Course Objectives (II)

- **Tentative:** Gauge transformations, canonical momentum.
- **Hydrogen** atom.
- **Density matrix**, statistical mixtures, decoherence.
- **Entanglement**.
- Feynman **path integral** approach.



Course Work

- **Problem sets:** Weekly.
- **Participation:** class attendance, classroom discussion, quizzes.
- **Midterms:** Two midterm tests planned (with possibly a 3rd).
- **Final exam:** December 7, 2–5 pm.

Problem sets	15 %
Participation	15 %
Midterm	40 %
Final Exam	30 %
Total	**100 %**

Problem Sets

You should complete the problem sets on your own.

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Allowed

- <u>Verbal discussion</u> of problems between students.
- <u>Sketch-level</u> blackboard-whiteboard discussion with other students.
- Ask instructors for assistance during office hours.
- Consultation of book/journal references (and internet).

Problem Sets

You should complete the problem sets on your own.

Allowed

- Verbal discussion of problems between students.
- Sketch-level blackboard-whiteboard discussion with other students.
- Ask instructors for assistance during office hours.
- Consultation of book/journal references (and internet).

Not Allowed (i.e., honor code violation)

- In-depth blackboard-whiteboard collaboration with other students.
- Consultation of solution manuals.
- Artificial intelligence generated solutions (e.g., ChatGPT, Claude, etc).

References

Text: The majority of the course materials and problem sets will be taken from the following required text for the course:

Modern Quantum Mechanics, by J. J. Sakurai and J. Napolitano. (3rd ed., 2021)

Some course materials will also be taken from the following texts:

Quantum Mechanics, by C. Cohen-Tannoudji, B. Diu, and F. Laloë (1st ed., 1992).

Quantum Mechanics, by L. I. Schiff (3rd ed, 1968).

Quantum Mechanics, by L. D. Landau and E. M. Lifshitz (3rd ed., 2003).

Introduction to Quantum Mechanics, by D. J. Griffiths (1995).

Schedule (I)

Week 0: 8/27 **Review of basic quantum mechanics**
Foundational experiments, interference, superposition, particles & waves.

Week 1: 9/1-3 **Mathematical foundation of QM**
Vector spaces, Bra-Ket notation, Hilbert spaces, Hermitian operators.

Week 2: 9/8-10 **Postulates of QM**
Postulates of QM, wavefunctions, probabilities, q-p repr., Schrodinger eq.

Week 3: 9/15-17 **Observables, position, momentum, time, energy**
Generator of translations, generator of time evol., Hamiltonian, Heisenberg pict.

Week 4: 9/22-24 **QM basics: 2-state systems**
Eigenenergies, eigenstates, avoided crossings, time evolution, Josephson osc.

Week 5: 9/29-10/1 **Schrodinger equation in 1D**
Midterm #1, free particle, scattering by a barrier, tunneling, square well potential.

Schedule (II)

Week 6: 10/6 **Quantum harmonic oscillator**
Creation and annihilation operators, Foch states, coherent states.

------------------------------ Fall Mini-Break --------------------------------

Week 7: 10/13-15 **Angular Momentum 1**
Orbital angular momentum, spin, commutators, raising & lowering operators.

Week 8: 10/20-22 **Angular momentum 2**
Generator of rotations, spherical harmonics, Euler angles.

Week 9: 10/27-29 **Addition of angular momenta**
Addition of operators, Clebsch-Gordan coefficients.

Week 10: 11/3-5 **Schrodinger equation in 3D**
Central potential, spherical square well, hydrogen atom.

Week 11: 11/10-12 **Hydrogen atom**
Midterm #2, Hydrogen atom, spin-orbit coupling, hyperfine Hamiltonian.

Schedule (III)

Week 12: 11/17-19 **Particle+EM Field OR Identical Particles (tentative)**
Gauge transformations, canonical momentum, bosons and fermions.

Week 13: 11/24 **Density matrix and decoherence**
Entanglement, statistical mixtures, coherence, decoherence, density matrix.

-------------------------------- Thanksgiving Break --------------------------------

Week 14: 12/1-3 **Feynman path integral formulation**
Propagators, Feynman path integral picture, semi-classical picture of QM.

December 7, 2026, 2-5 pm **Final Exam**

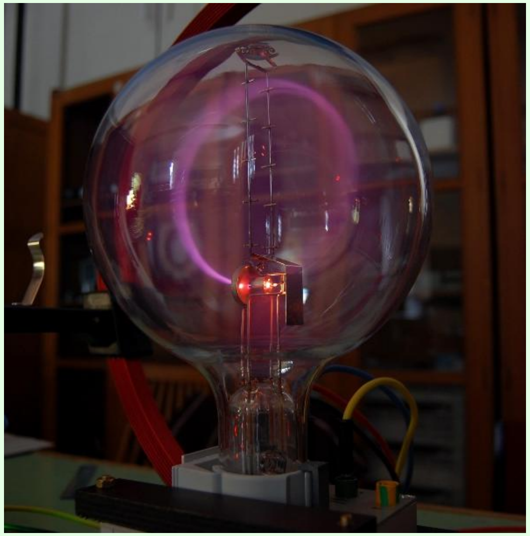
Quantum Accuracy

Electron's g-factor

Schrodinger: $g_e = 1.0$

Dirac: $g_e = 2.0$

QED: $g_e = 2.\underbrace{002\ 319\ 304\ 362}_{\text{12-digits}}$


[Wikipedia, 2009]

Theory and experiment agree to 9 digits.

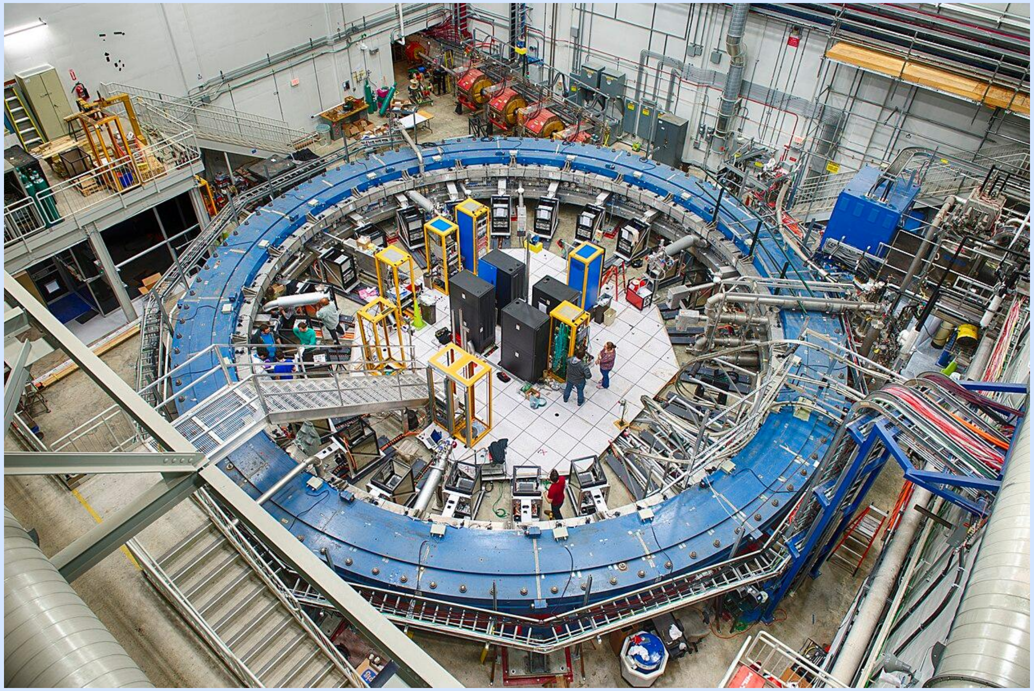
Quantum Accuracy – *again !!*

Muon’s g-factor

Schrodinger: $g_e = 1.0$

Dirac: $g_e = 2.0$

QED: $g_e = 2.\underbrace{000\ 023\ 318\ 414}_{\text{12-digits}}$



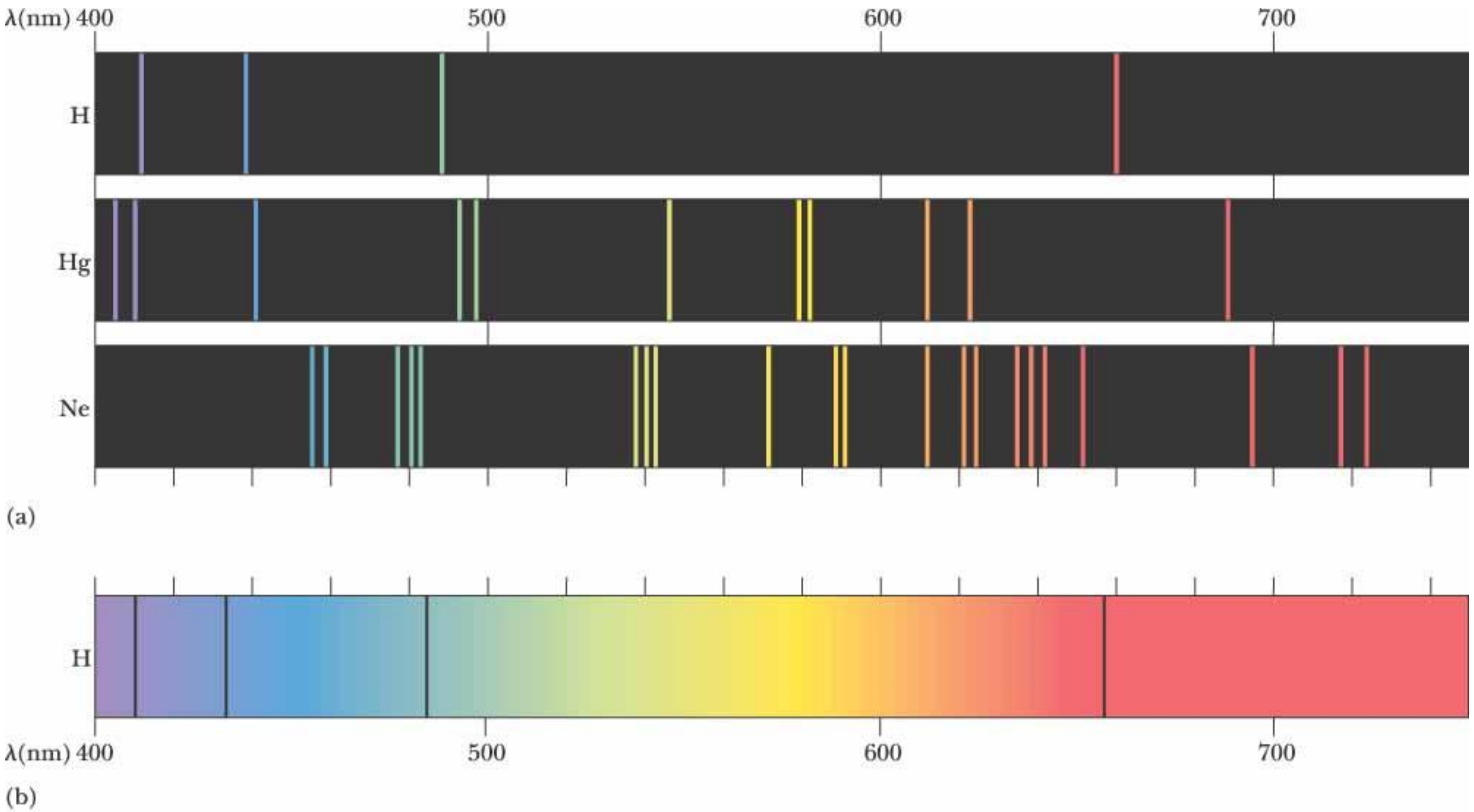
[Photo: FermiLab]

muon g-2 experiment

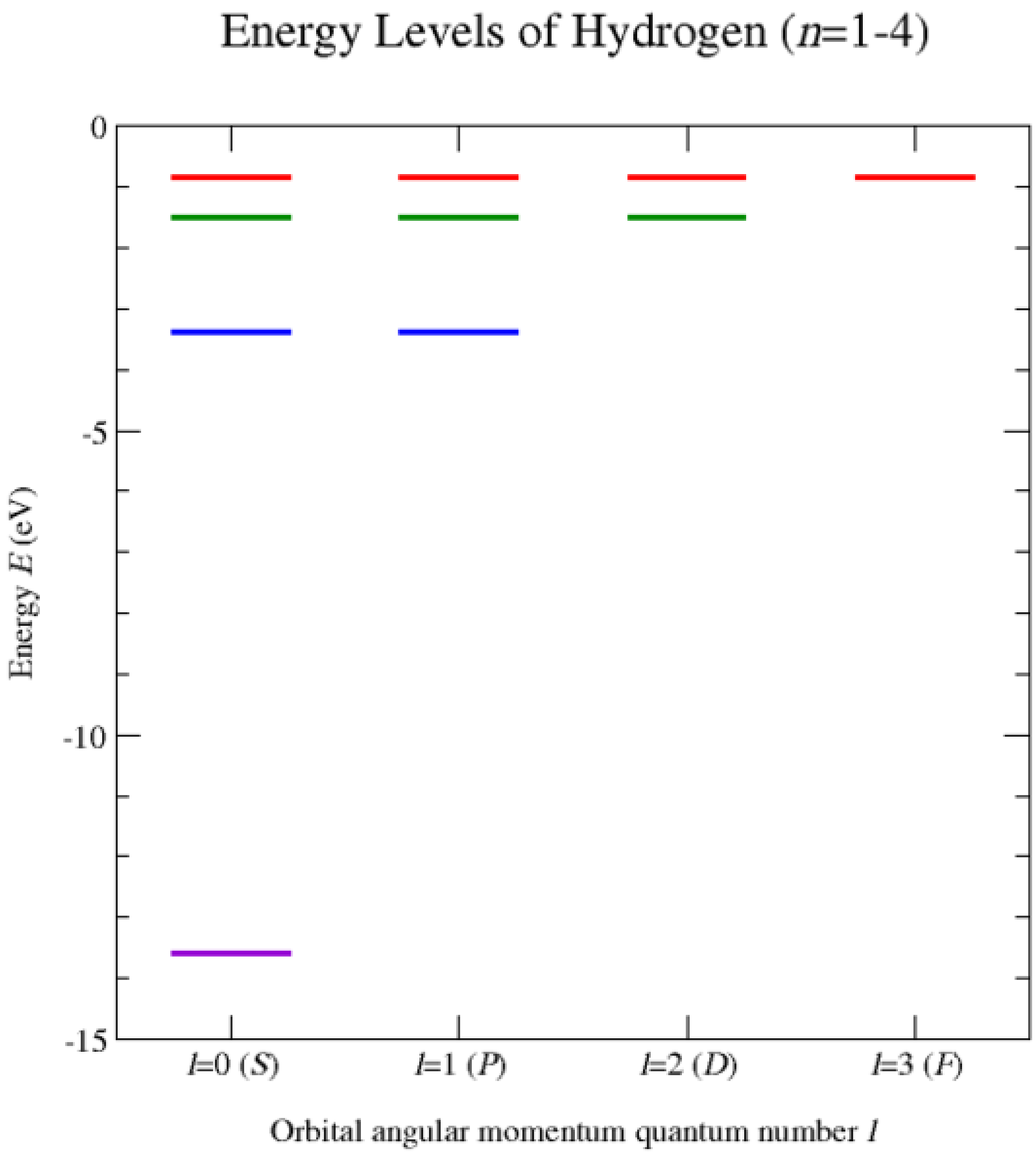
Theory and experiment agree to 11 digits !!

How was quantum mechanics discovered?

Atomic Emission and Absorption Spectra

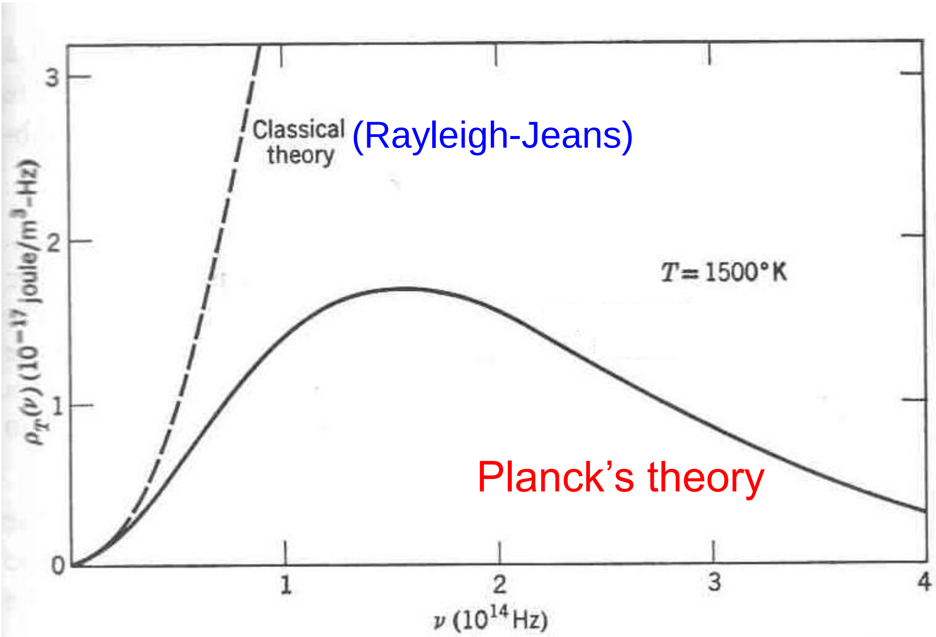


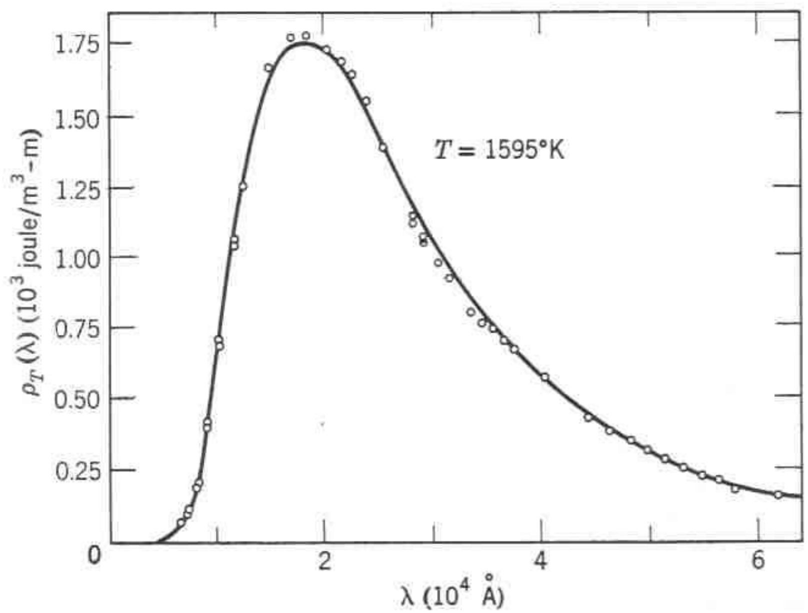
Quantum Version of Atoms



[Figure from wikimedia.org]

Blackbody Radiation: Rayleigh-Jeans vs. Planck





Experiment vs. Theory
(Coblentz data, 1916)

[figures adapted from ***Quantum Physics*** by Eisberg and Resnick, 1985.]

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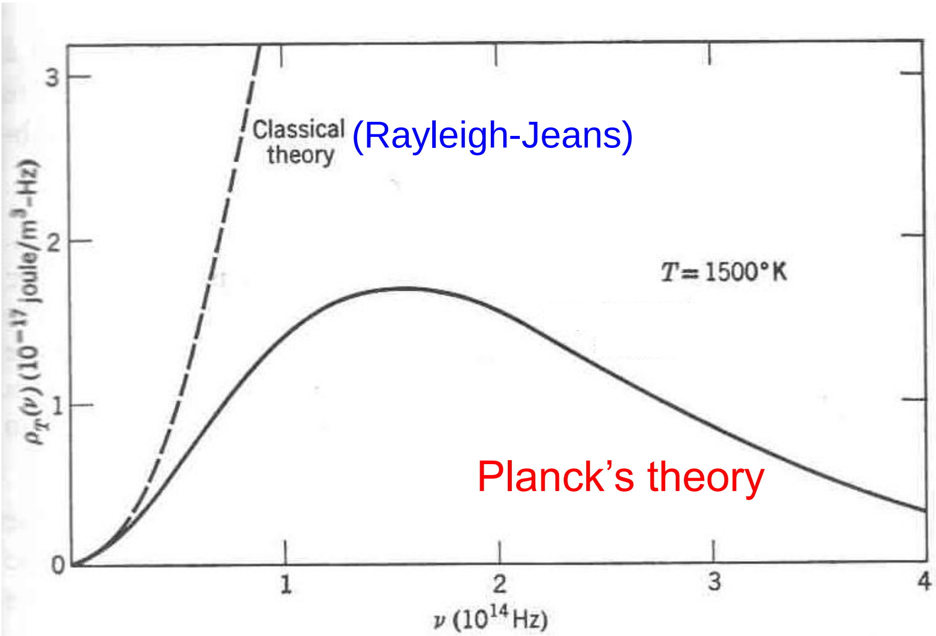

$$E = \hbar\omega$$

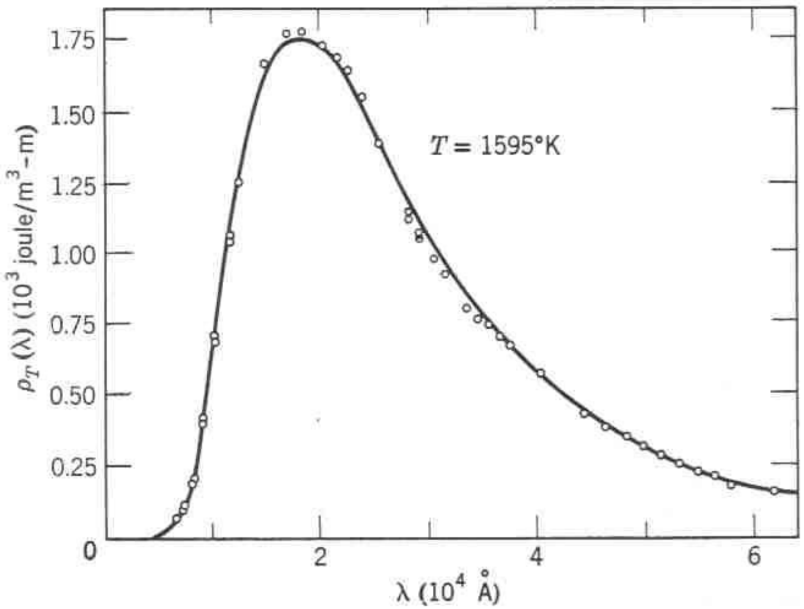

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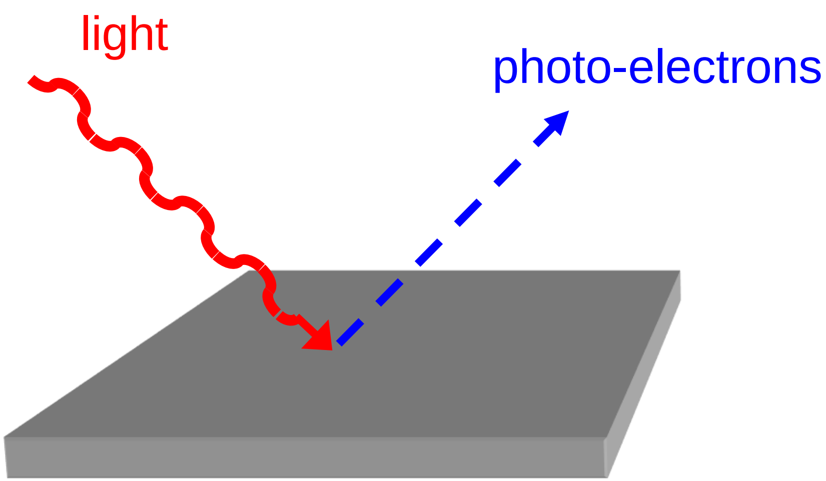
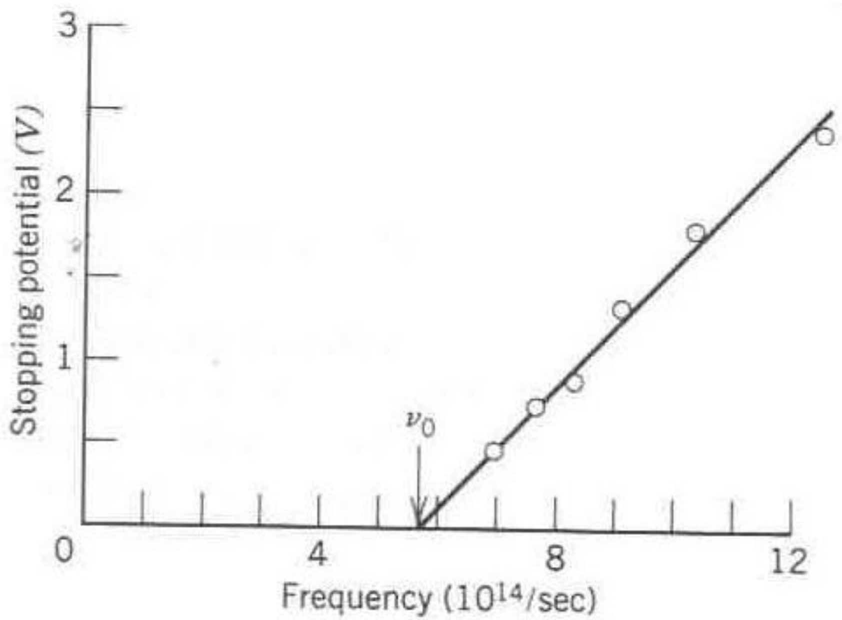
Photo-Electric Effect





Millikan's photo-electric data for sodium (1914)

[figure adapted from ***Quantum Physics*** by Eisberg and Resnick, 1985.]

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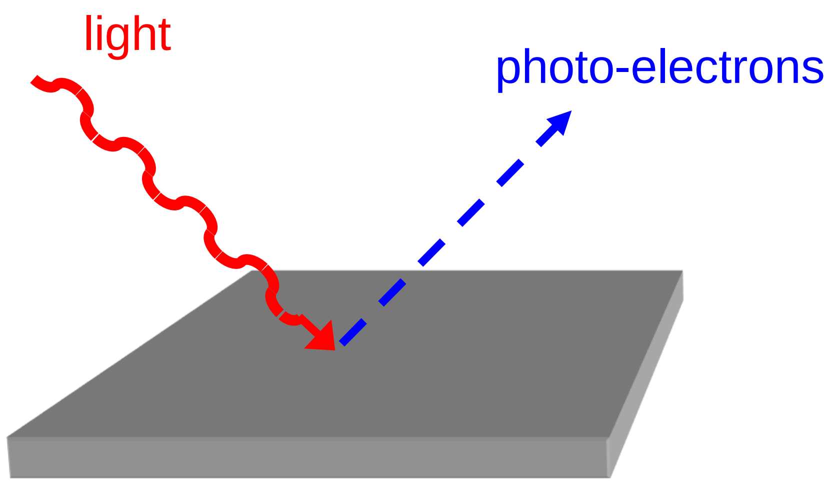

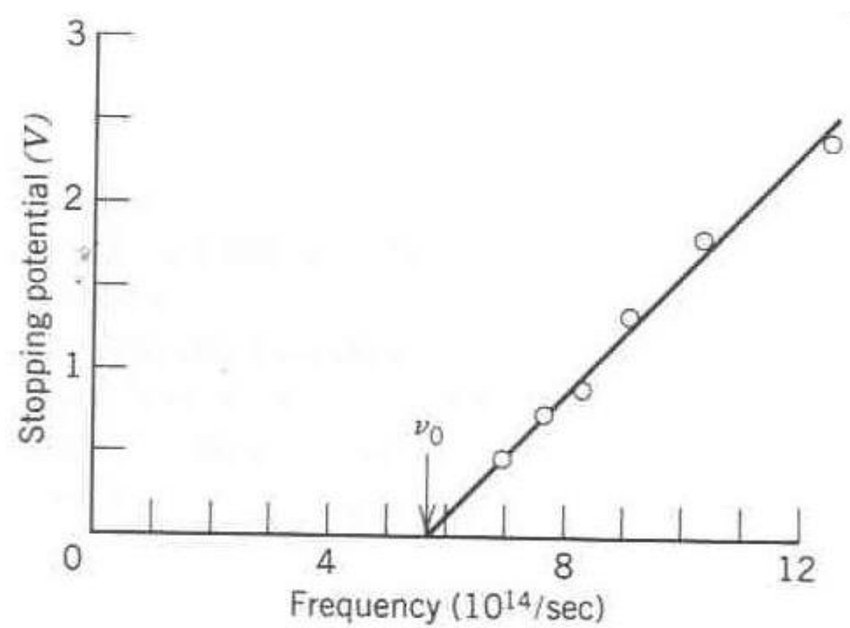

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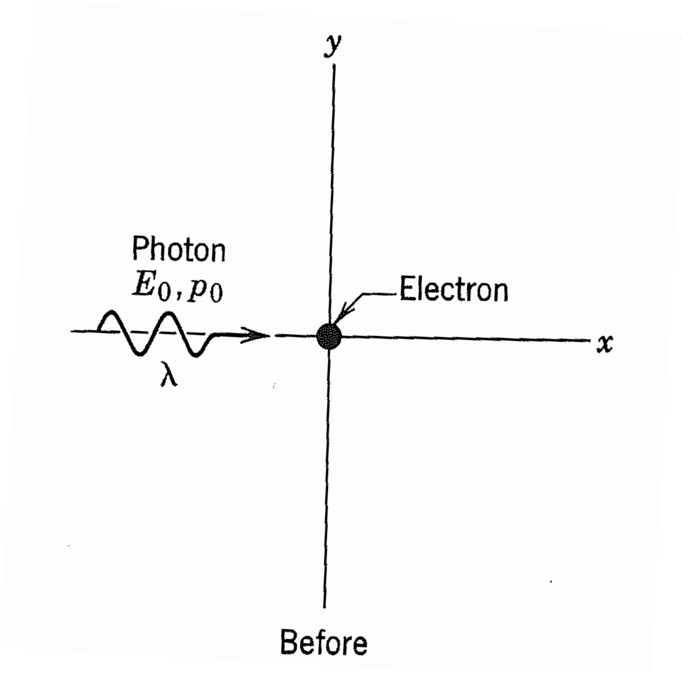
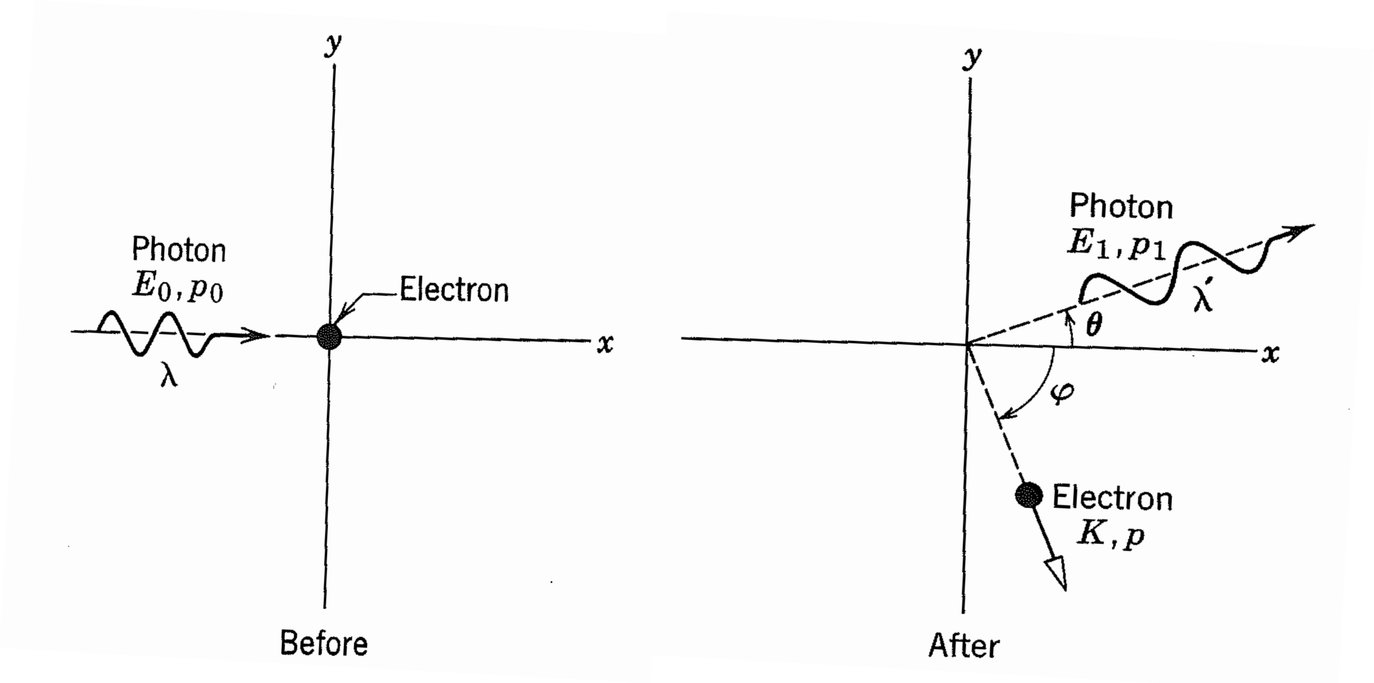
Compton Scattering



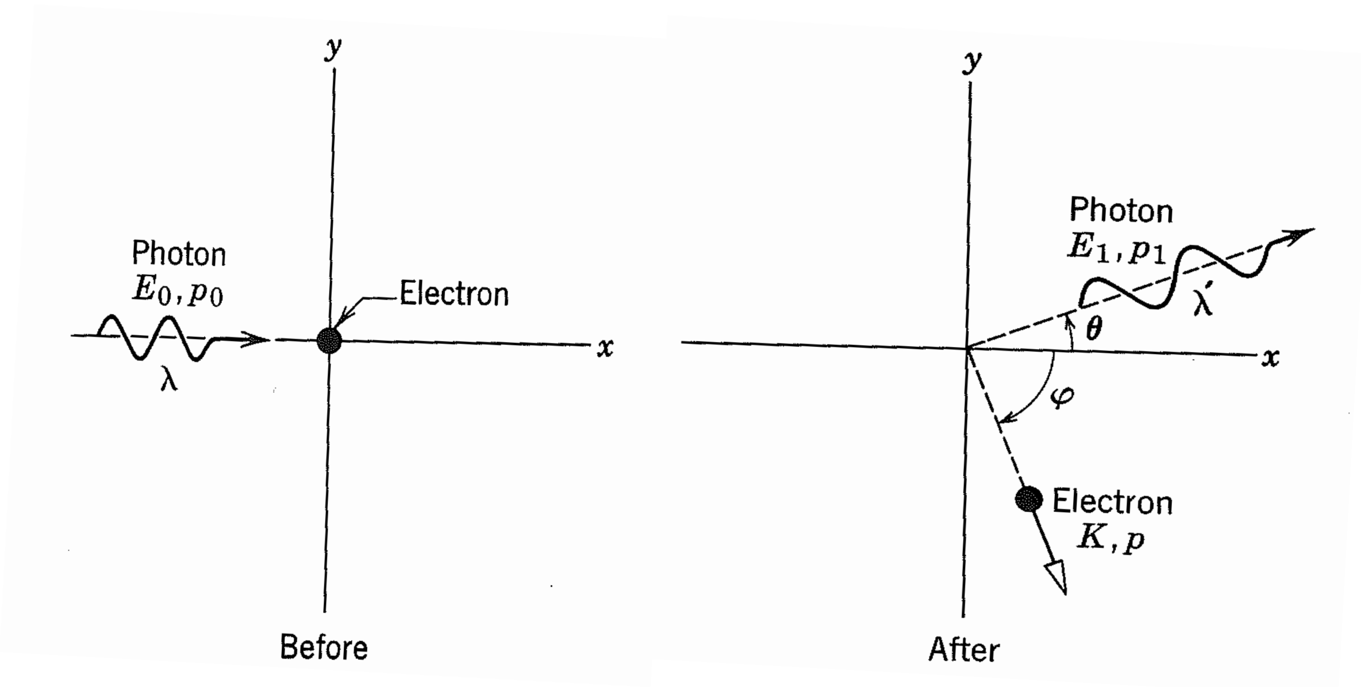
Compton Scattering



$$E = \hbar\omega$$

$$\vec{p} = \hbar\vec{k}$$

Compton Scattering



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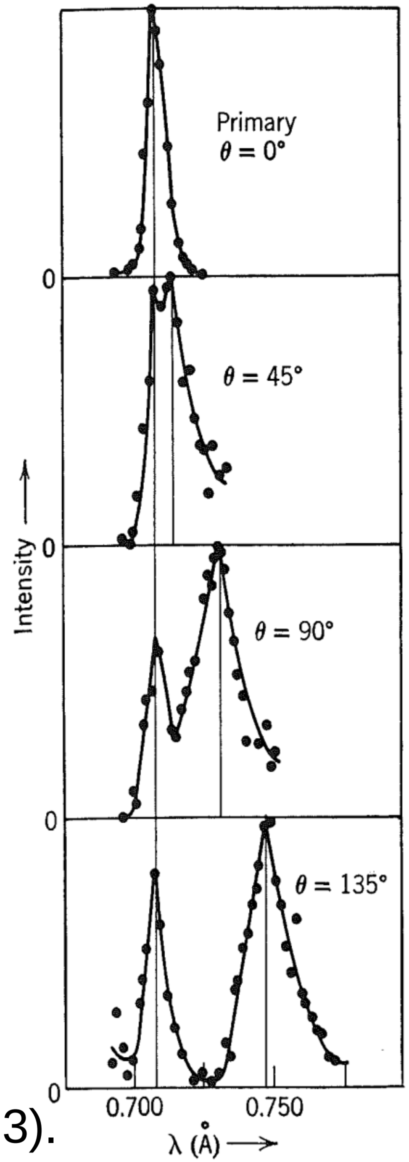
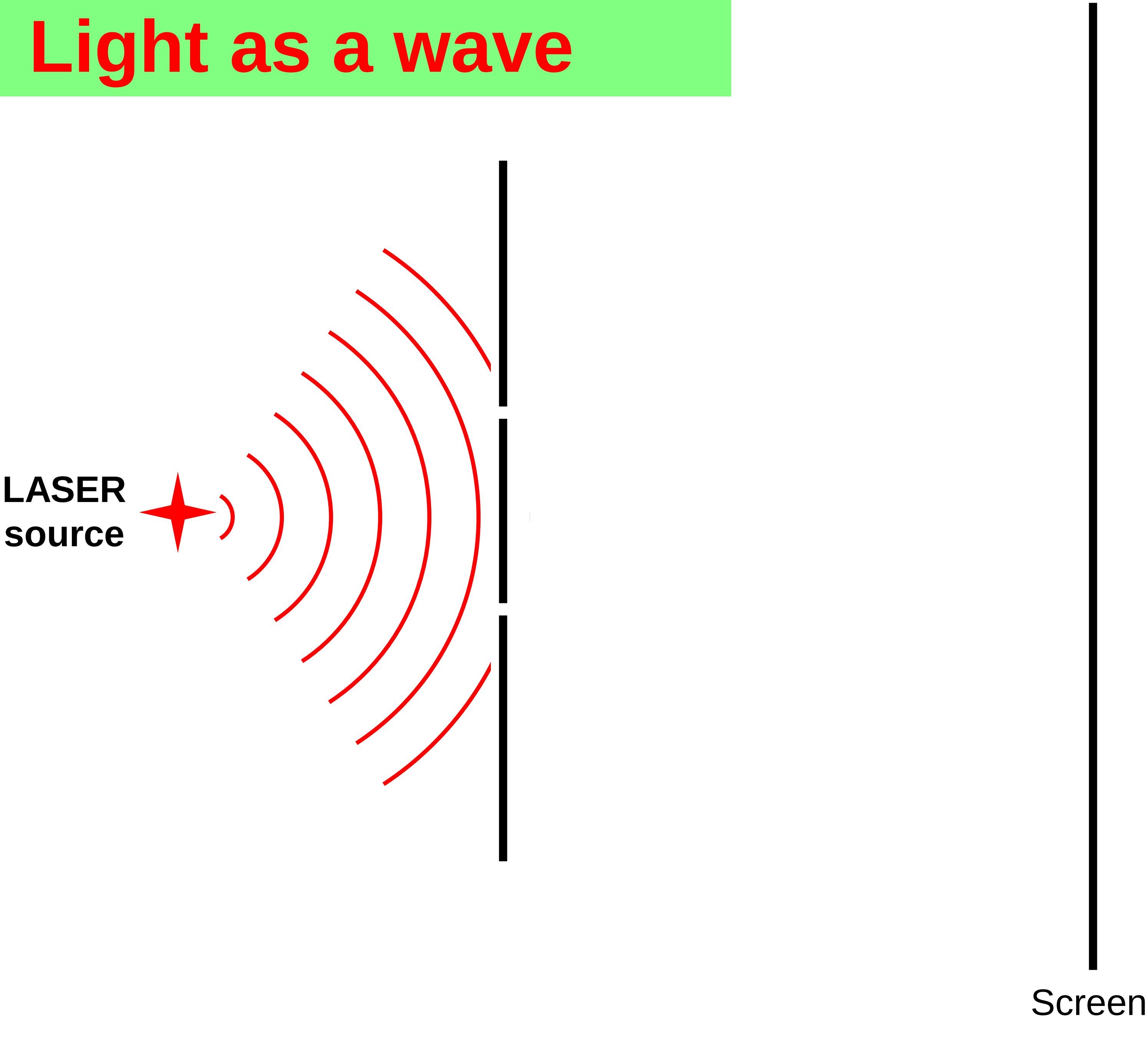


Compton's data for x-ray scattering in graphite (1923).

[figures adapted from ***Quantum Physics*** by Eisberg and Resnick, 1985.]

Quantum Mechanics
and
the double-slit experiment

Light as a wave



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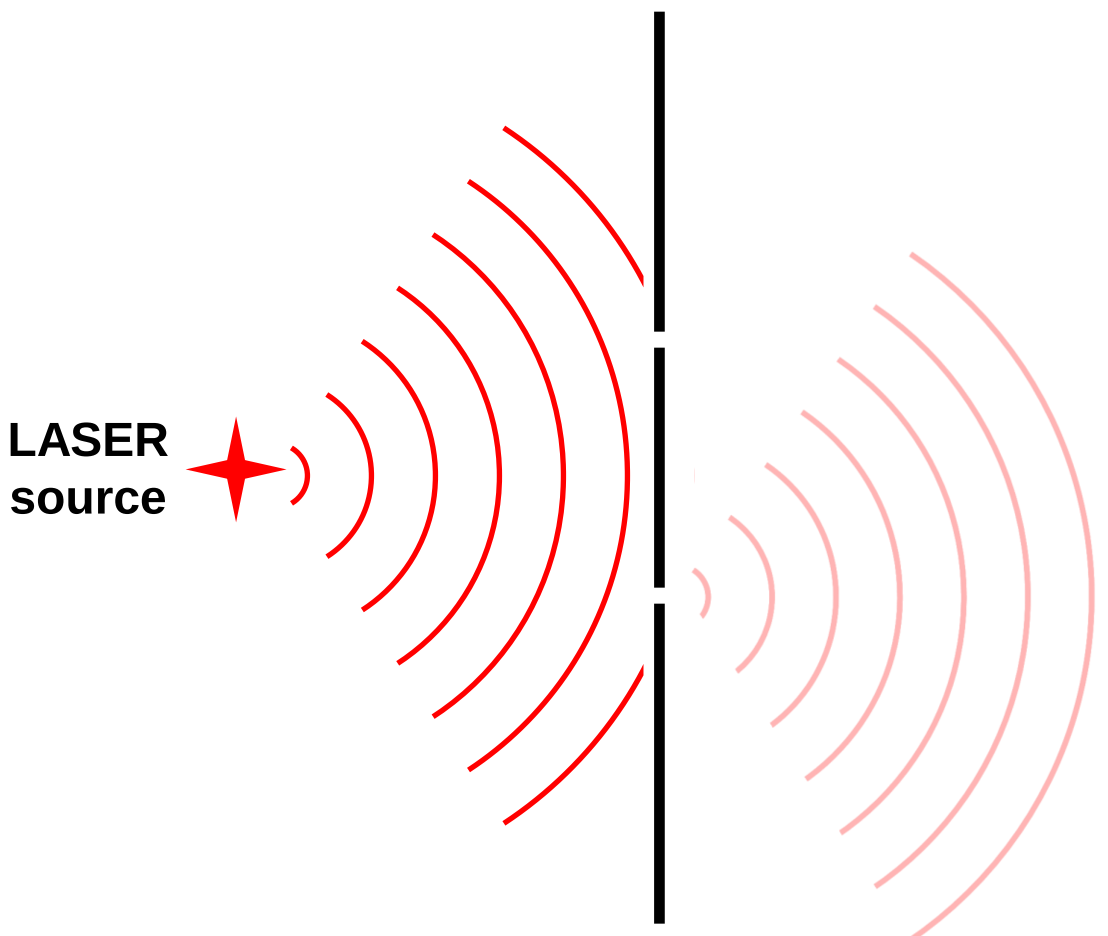
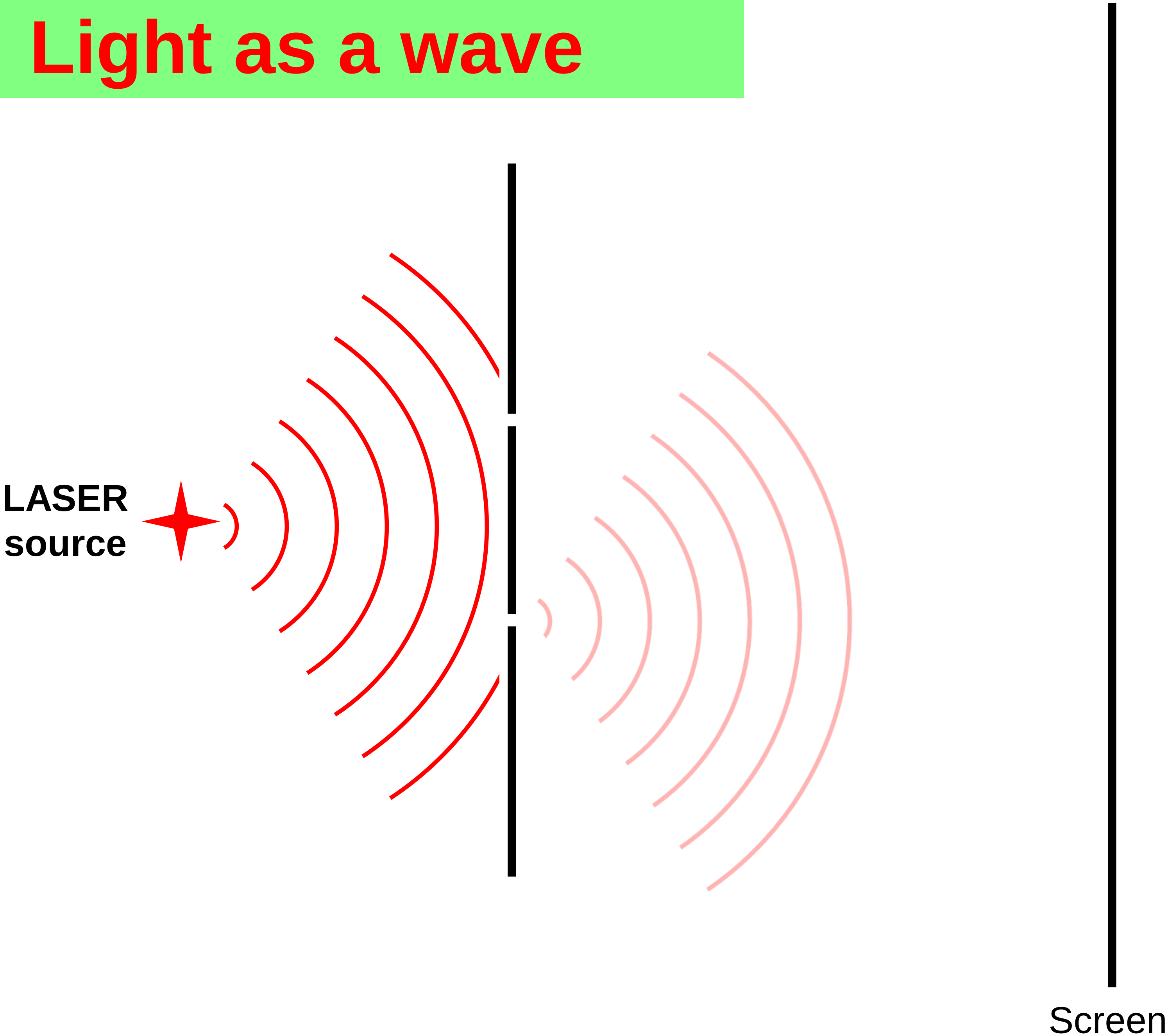


Light waves **diffract** as they go through the slits

Screen

Light as a wave



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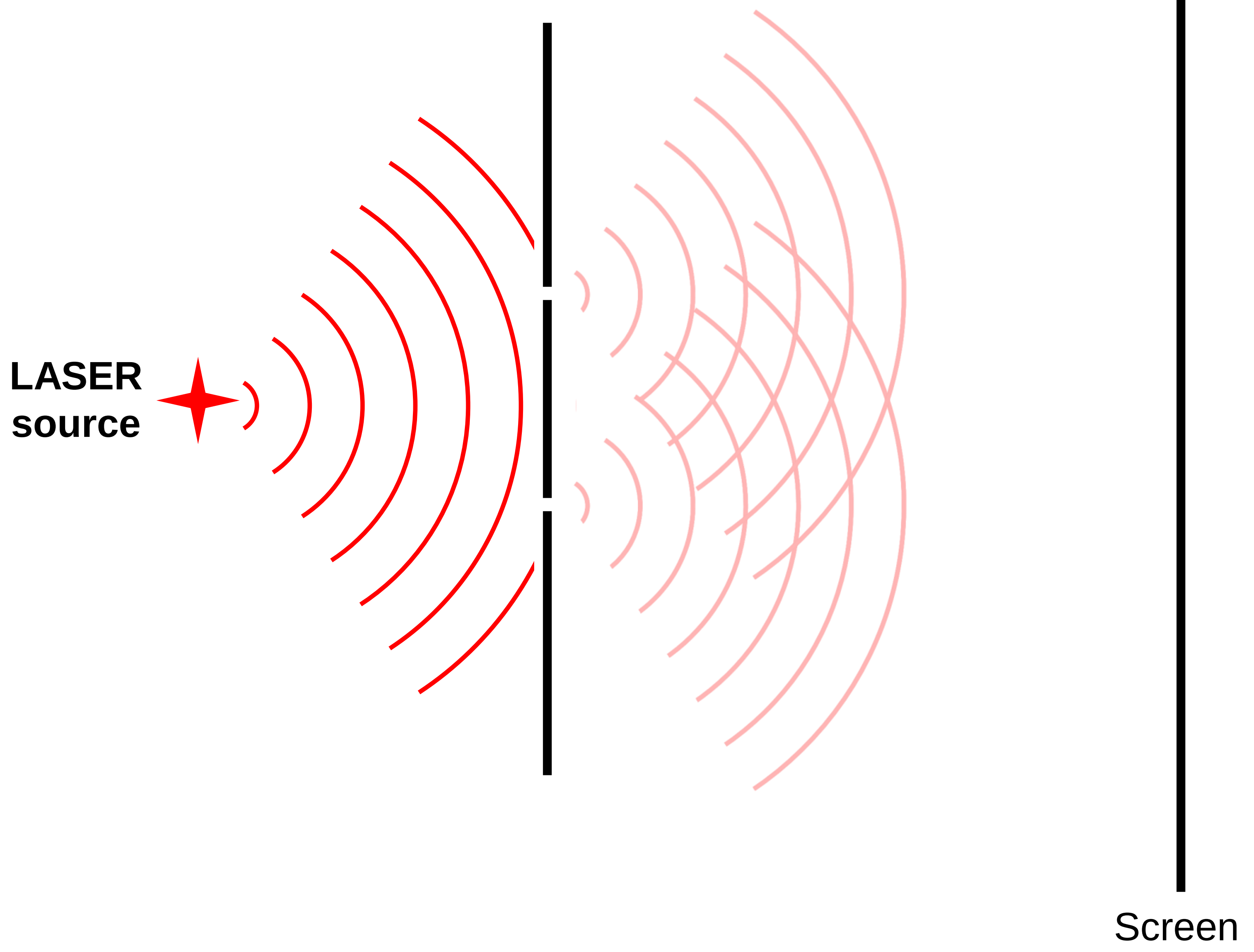

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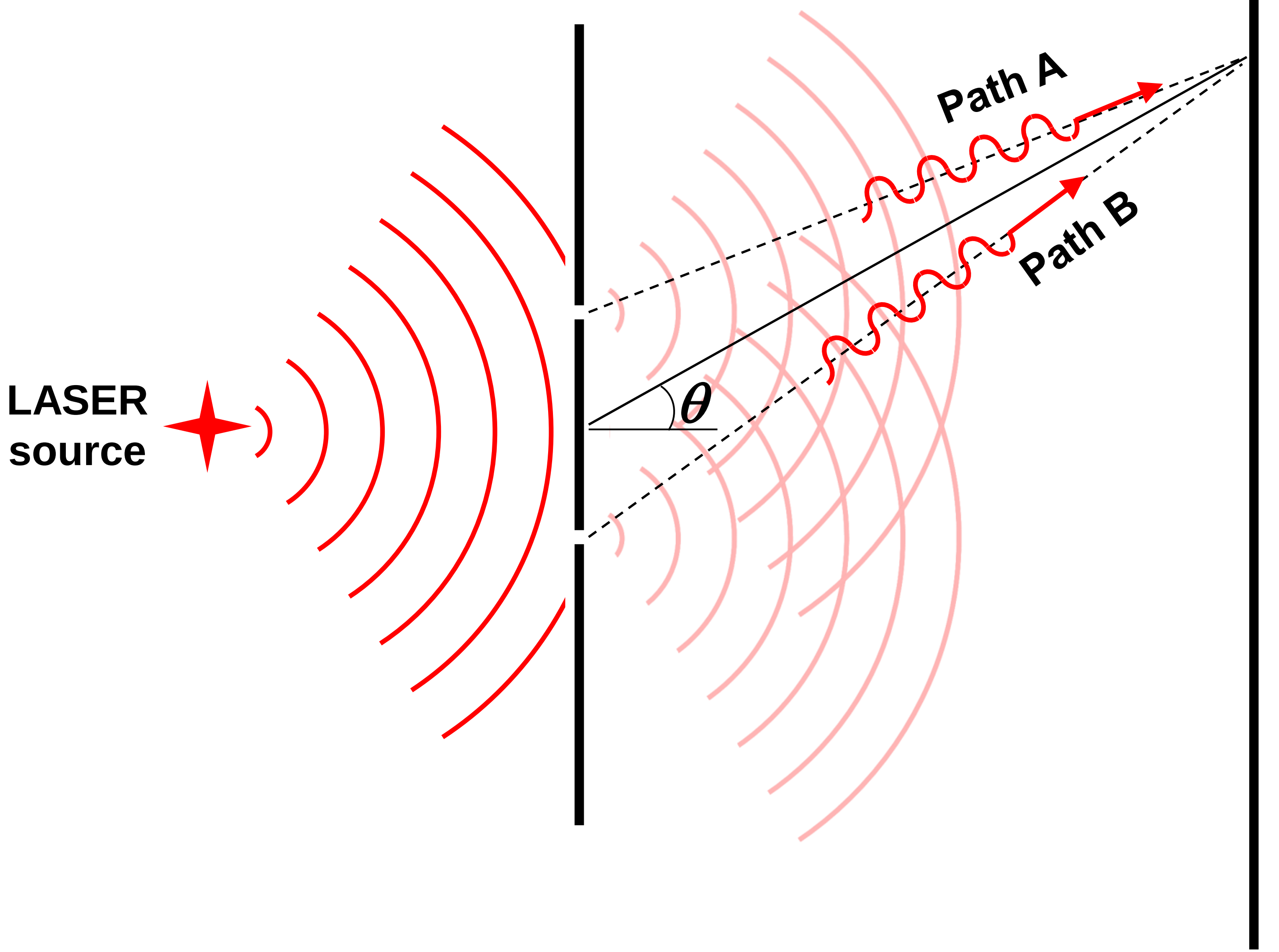

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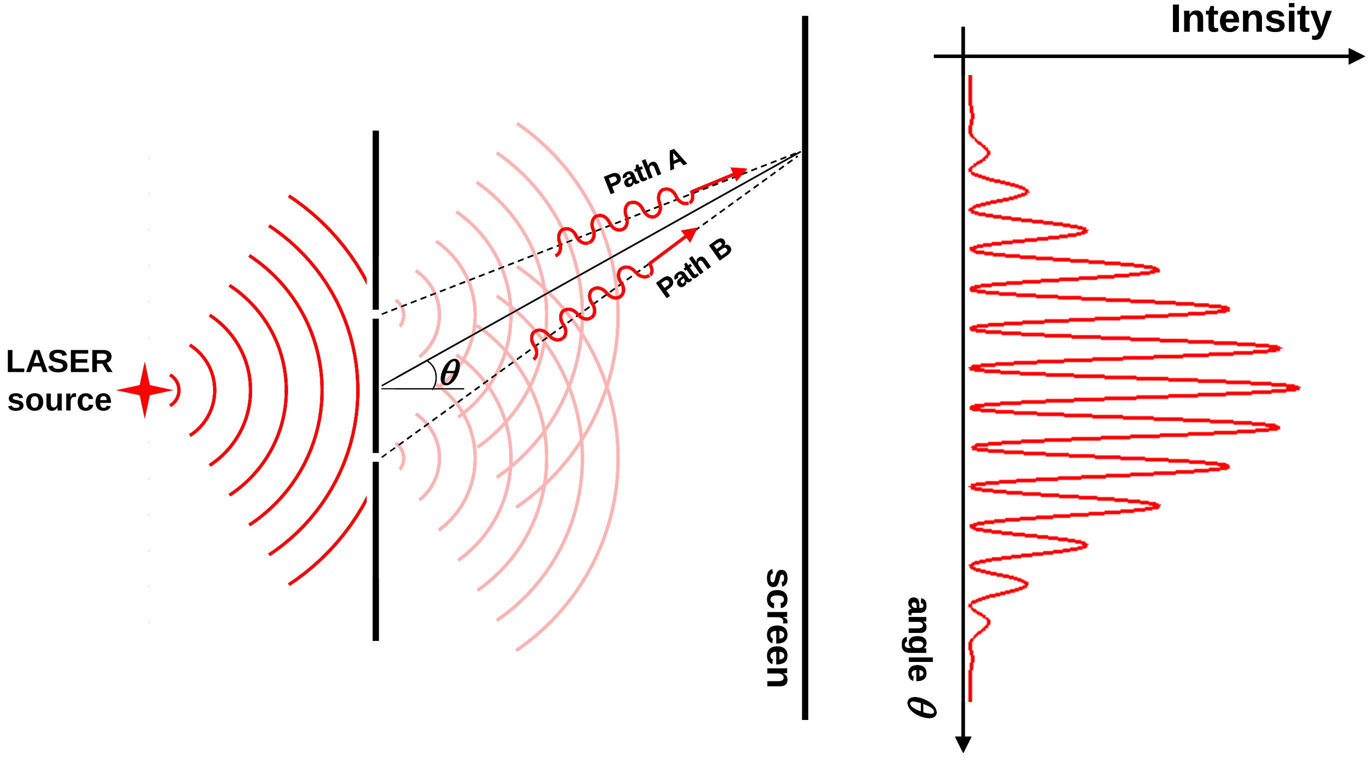
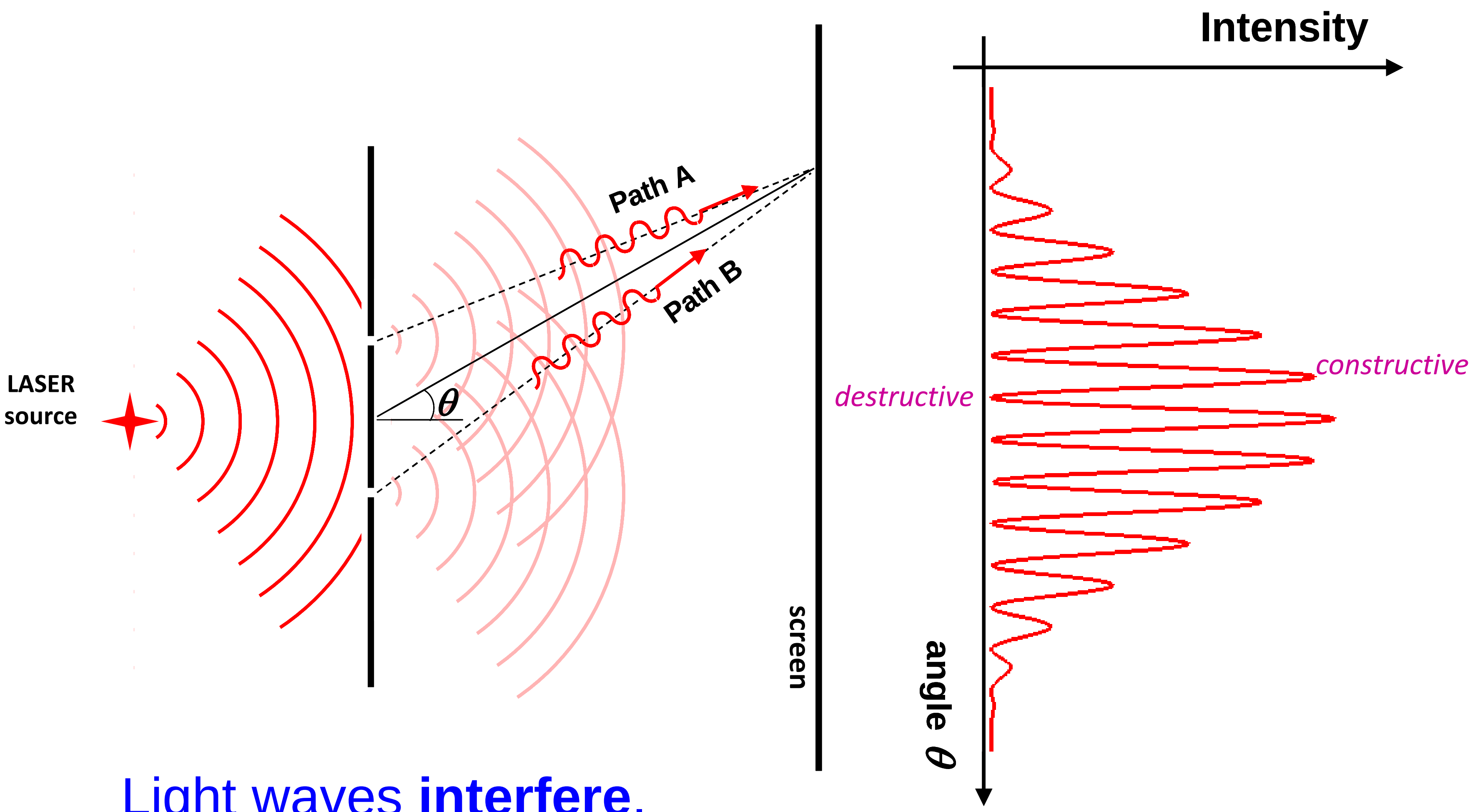


Light waves **interfere**.

Interference Experiment



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[Young's double slit experiment, by Thomas Young (1773-1829) in 1801-1803.]

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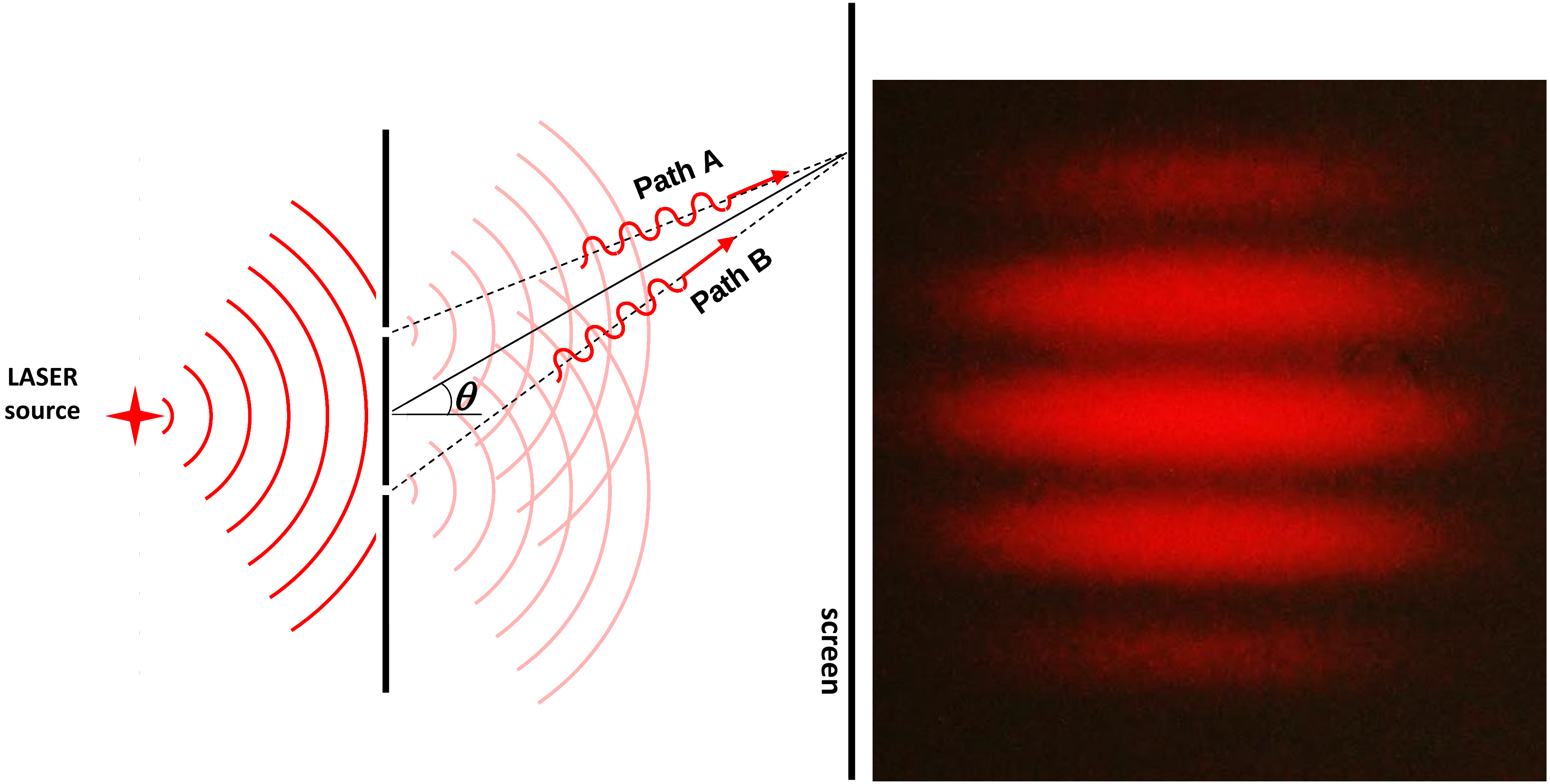
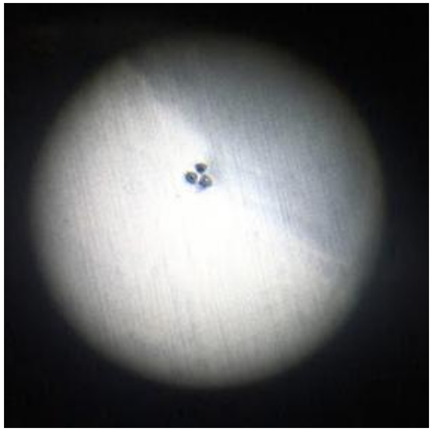
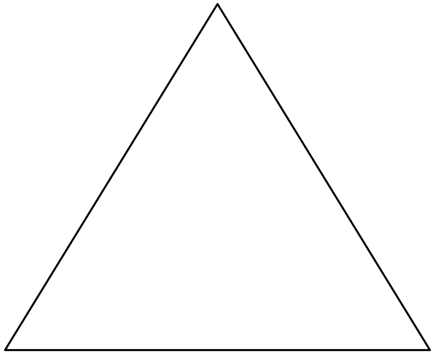


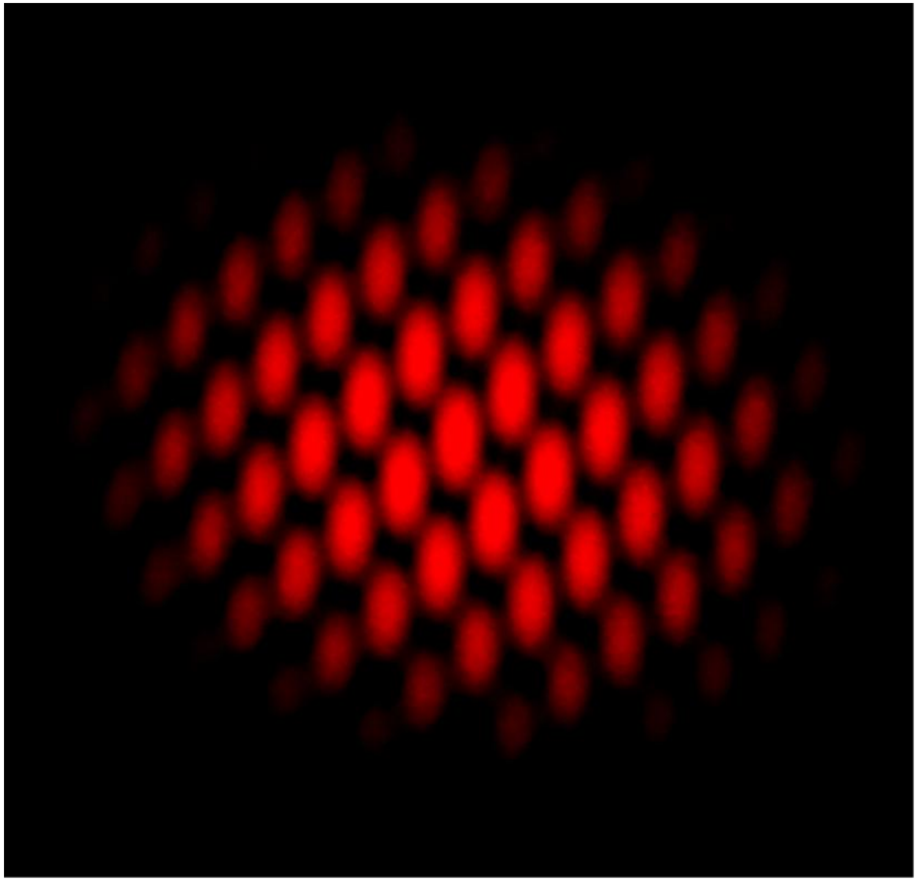
[data by M. Frayser, W&M 2018]

[Young's double slit experiment, by Thomas Young (1773-1829) in 1801-1803.]

3 Holes

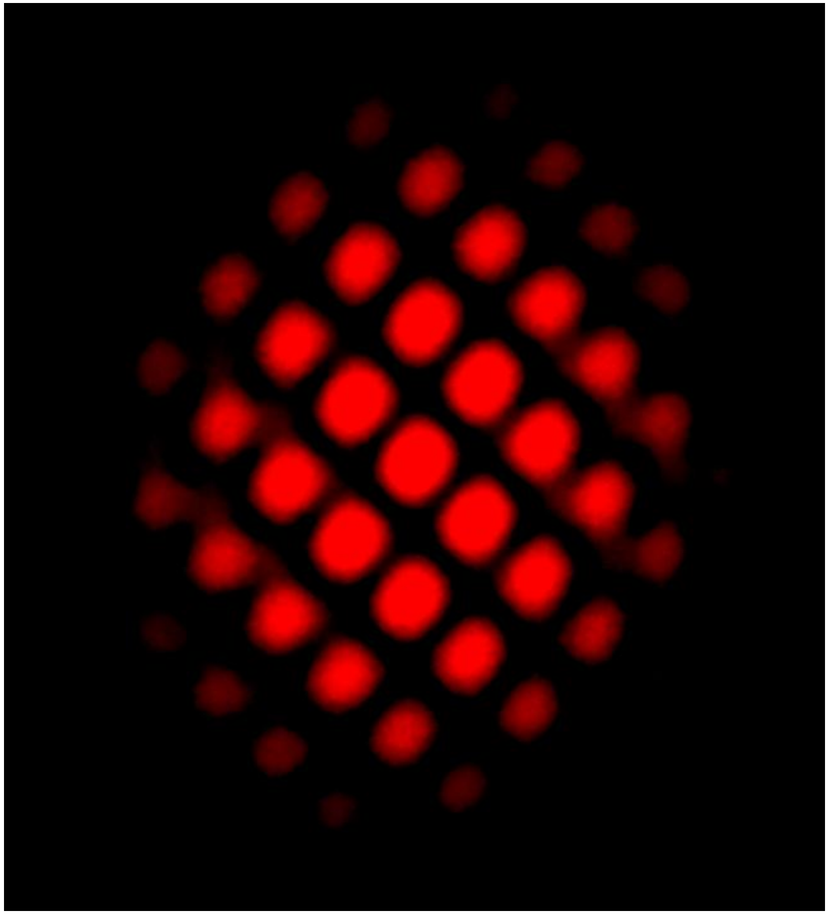


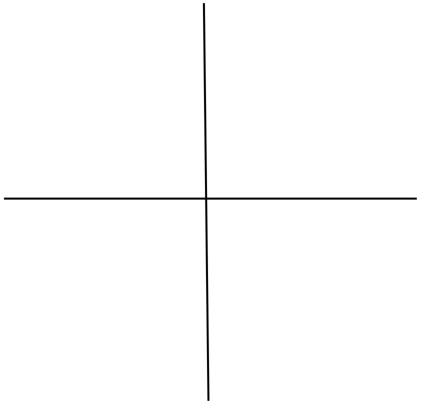




[figures from M. Frayser, W&M senior thesis, 2019]

4 Holes





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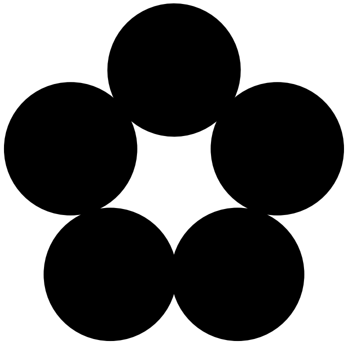
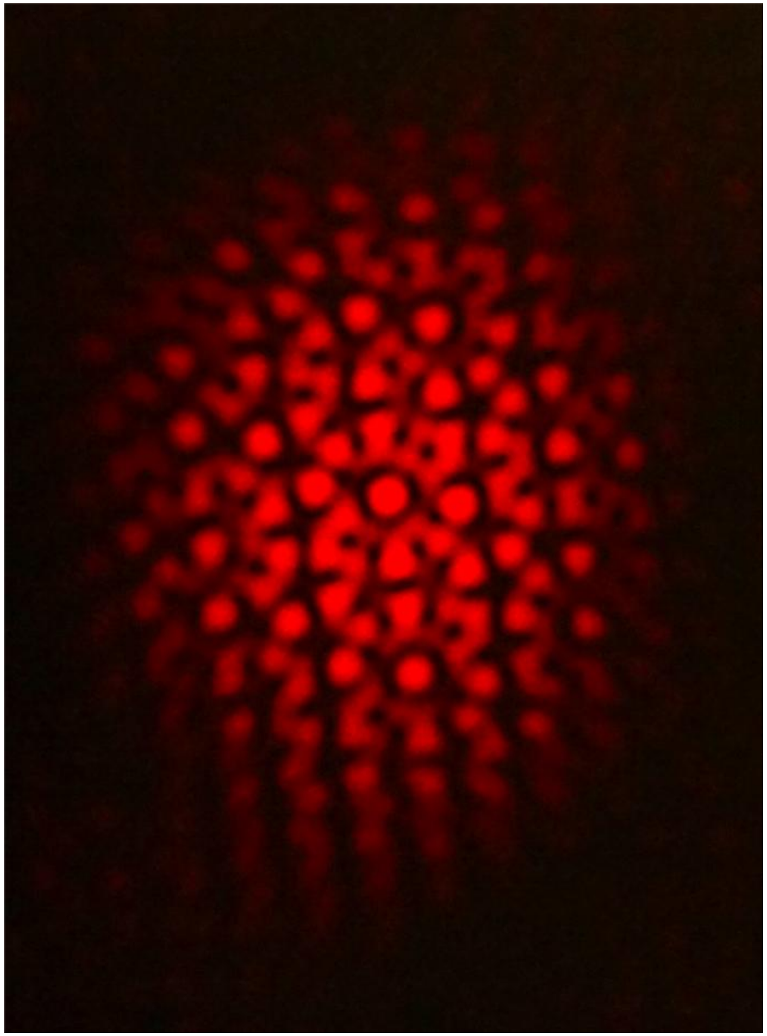
5 Holes

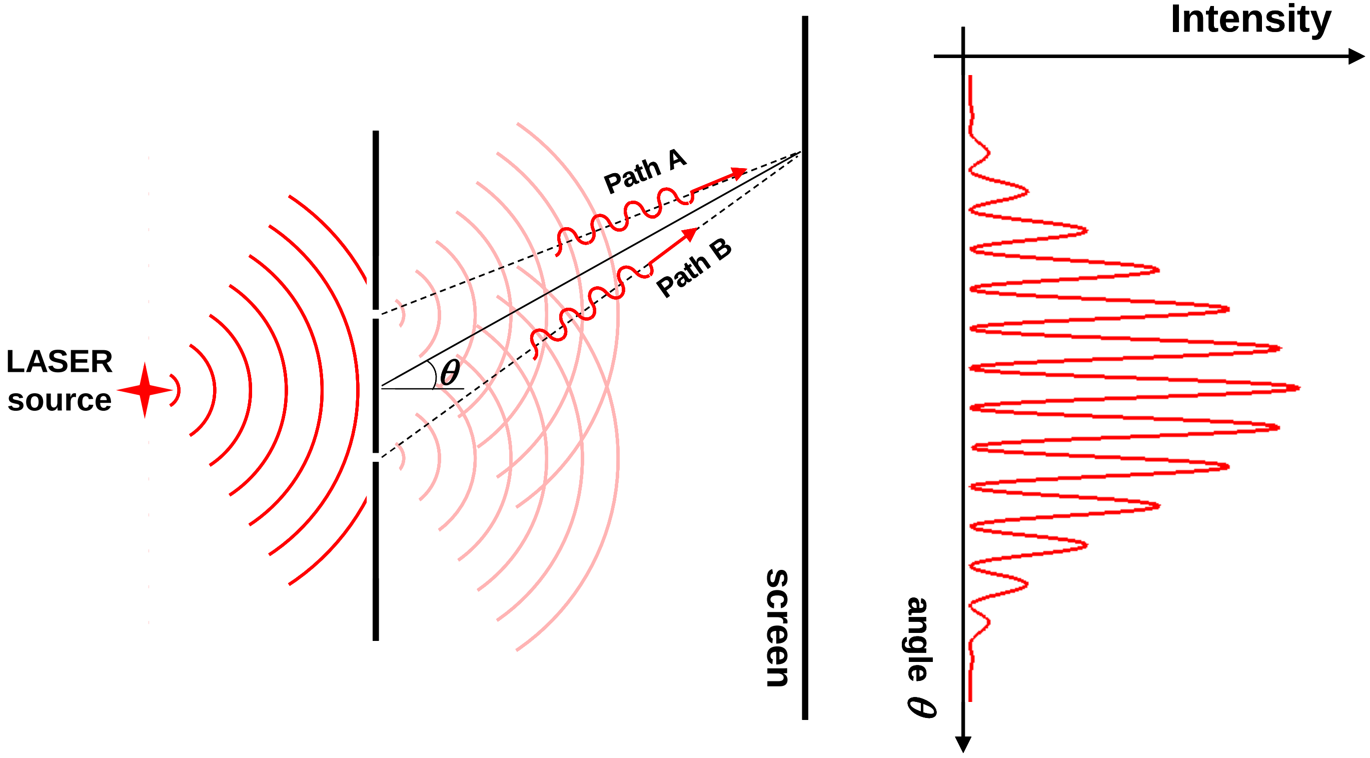




Quasi-crystal pattern → pattern does not truly repeat!!

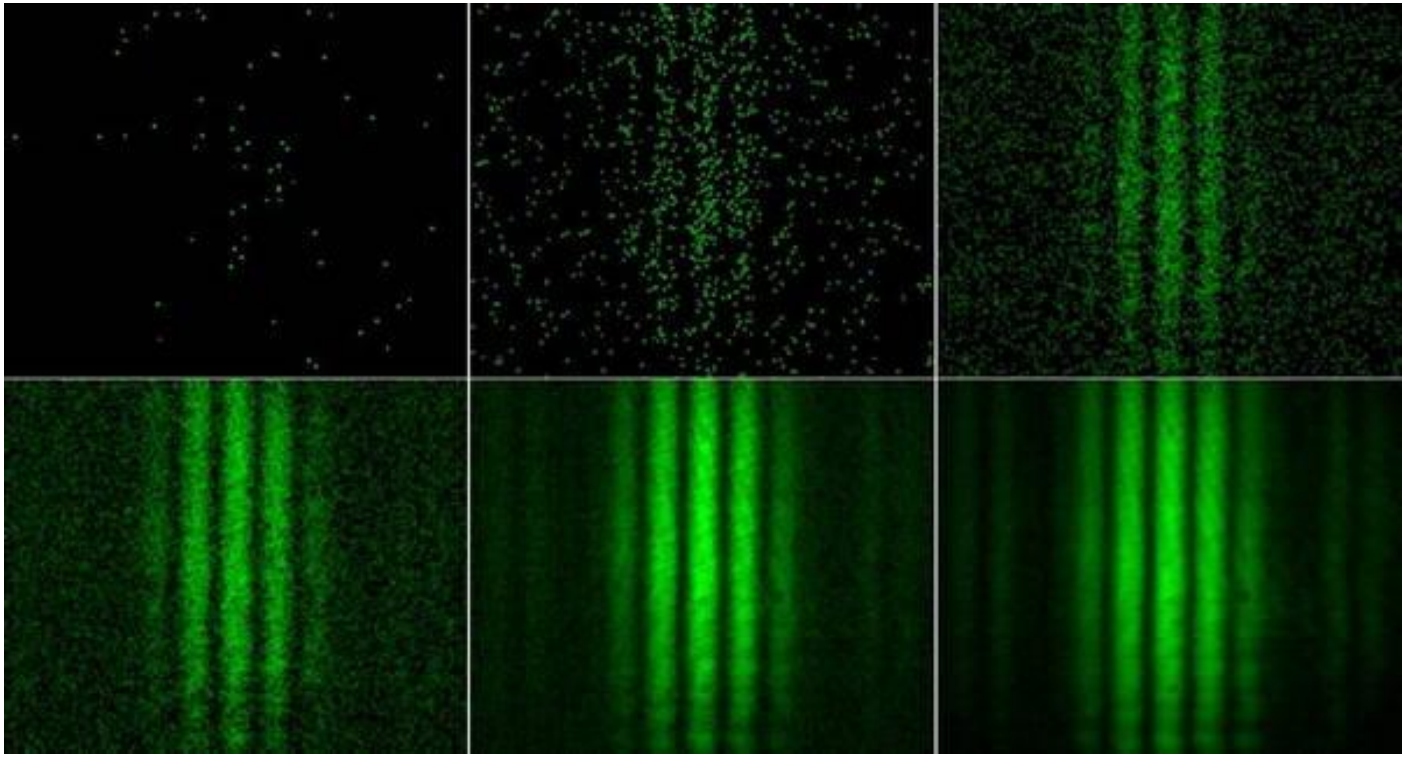
[figures from M. Frayser, W&M senior thesis, 2019]

Light as a wave



Light waves **interfere**.

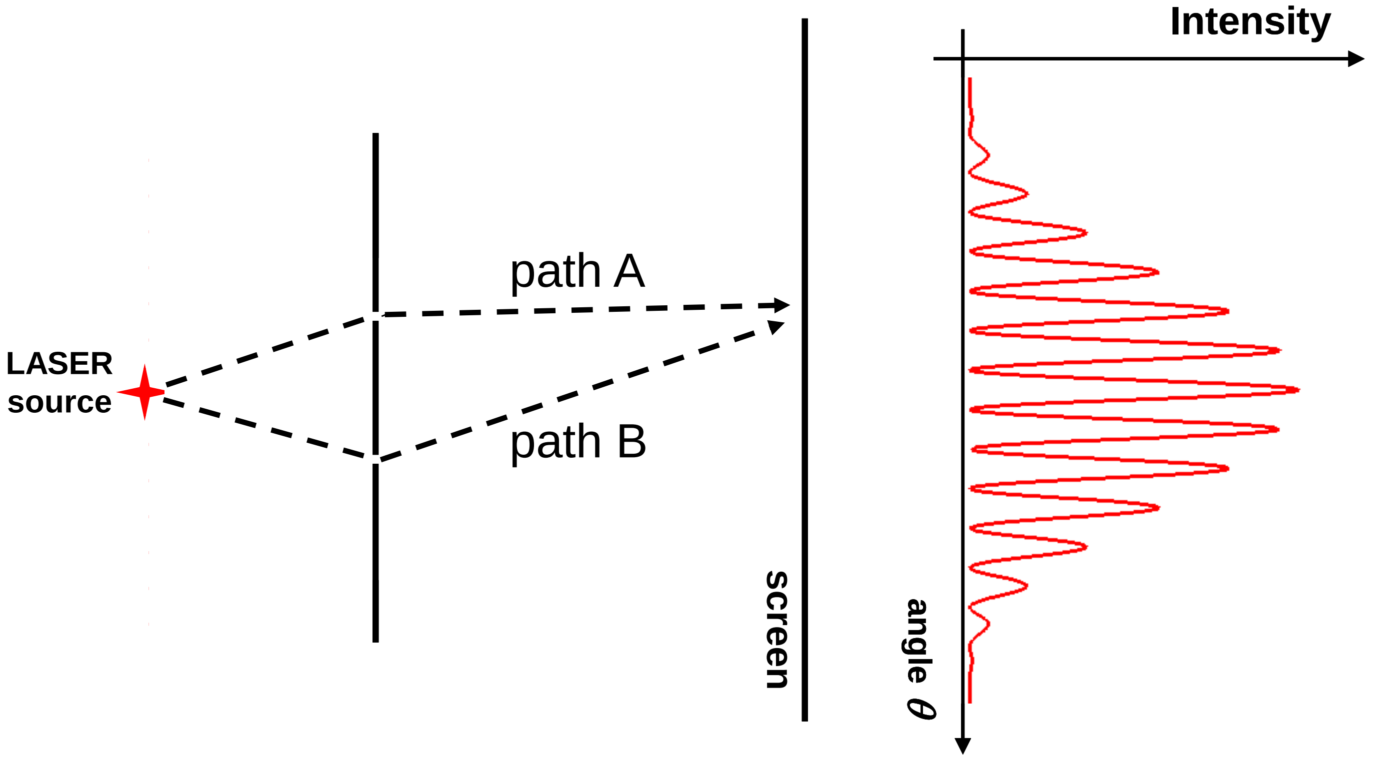
Also works for single photons !!!



[A. L. Weiss and T. L. Dimitrova, Swiss Physics Society, 2009.]

Experiment uses a CCD camera (i.e. sensor in your digital camera).

Photons follow 2 paths simultaneously



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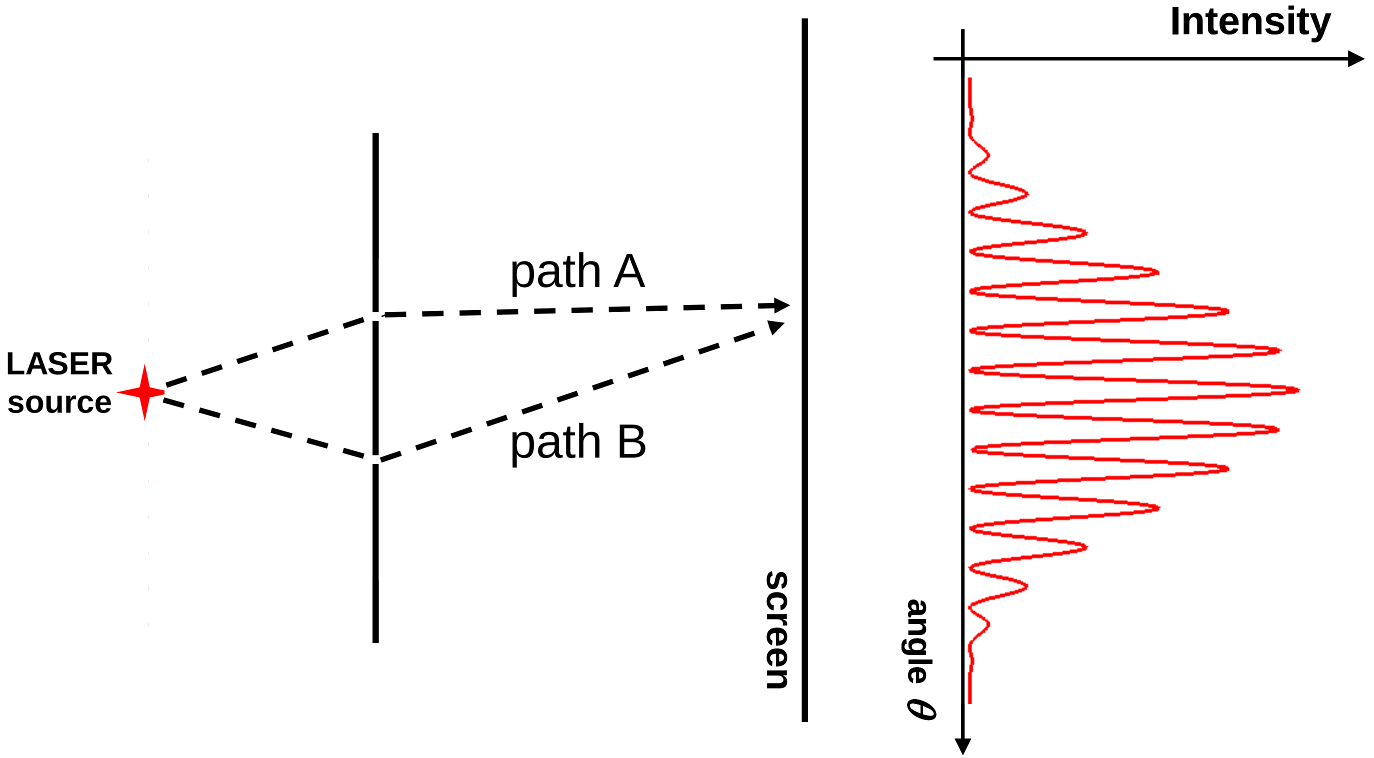
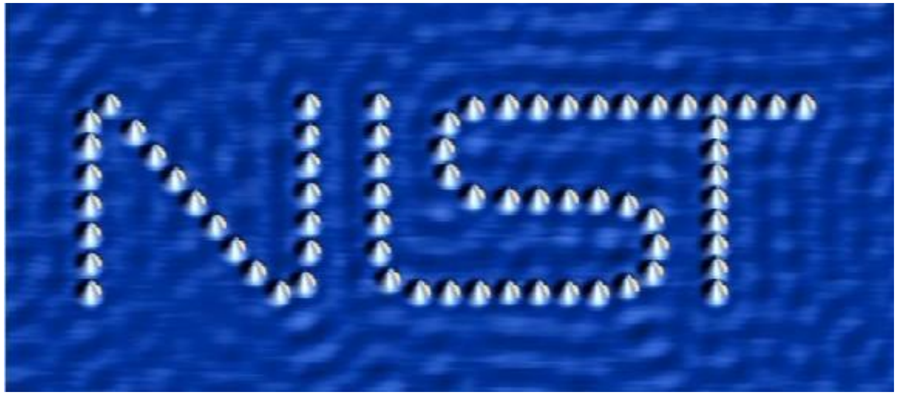


$$|\psi\rangle_{photon} = |A\rangle + e^{i\phi}|B\rangle$$

LIGHT IS A

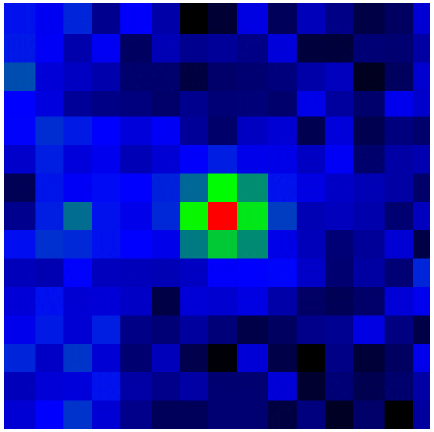
particle!

Atoms



Cobalt atoms on a copper surface
(scanning tunneling microscope image)

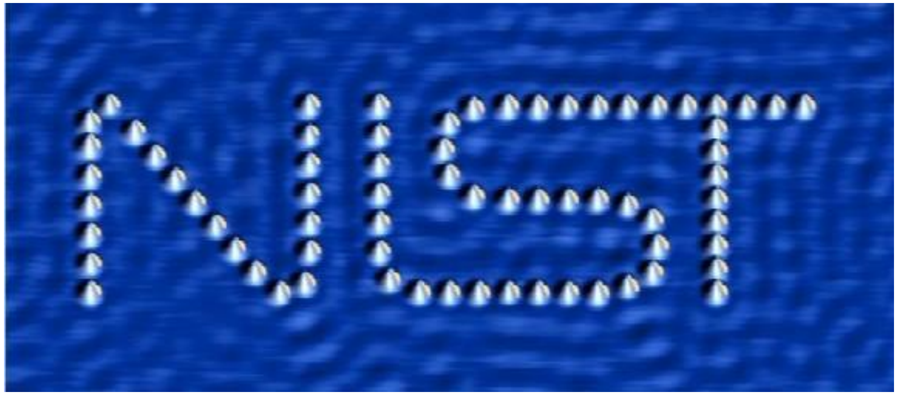
[image from www.nist.gov]



Single Rb atom
(laser cooled and trapped)

[image from Grangier group, www.optique-quantique.u-psud.fr]

Atoms



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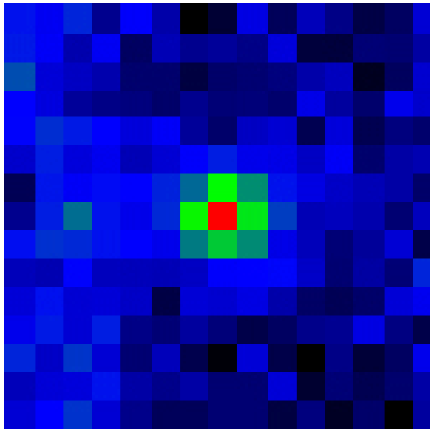

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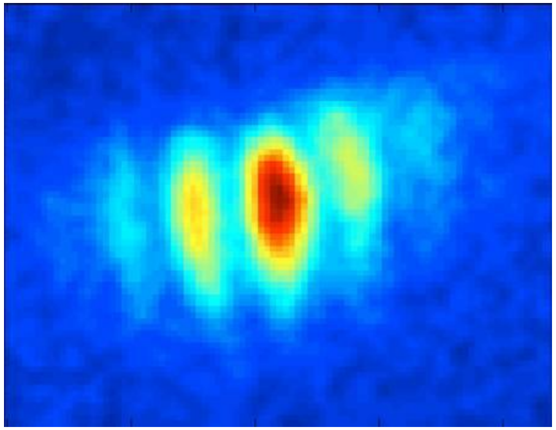

Interference of atoms in a Bose-Einstein condensate

Matter is also a particle!