Physics 621: Grad QM I
Thursday, September 10, 2026

Problem Set #1:
Review of Vector Spaces and Operators

Question 1: Triangle inequality
Consider two vectors $|u\rangle$ and $|v\rangle$ of a vector space V. Prove that $\|u+v\| \leq \|u\| + \|v\|$.

Question 2: (Cauchy-) Schwarz inequality
Consider two vectors $|u\rangle$ and $|v\rangle$ of a vector space V. Prove that $|\langle u|v\rangle| \leq \|u\| \, \|v\|$.

Question 3: Pauli spin matrices
(Sakurai & Napolitano problem 1.4, re-written here for clarity)

Suppose a 2×2 matrix X (not necessarily Hermitian, nor unitary) is written as

$$X = a_0 \mathbb{I} + \vec{\sigma} \cdot \vec{a} \, ,$$

where $\mathbb{I}$ is the identity matrix, a_0 and $\vec{a} = (\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3)$ are scalar numbers, and $\vec{\sigma} = (\sigma_\mathrm{x}, \sigma_\mathrm{y}, \sigma_\mathrm{z})$ are the Pauli matrices:

$$\sigma_\mathrm{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_\mathrm{y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_\mathrm{z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

a) How are a_0 and a_k (k=1,2,3) related to $tr(X)$ and $tr(\sigma_k X)$?

b) Obtain a_0 and a_k in terms of the matrix elements X_{ij}?

Question 4: Operator product and eigenvalues
Sakurai & Napolitano problem 1.9

Question 5: Simple 2×2 Hamiltonian
Sakurai & Napolitano problem 1.12

Question 6: Commuting operators and their eigenbasis
Sakurai & Napolitano problem 1.25

Question 7: Eigenvalues and a unitary operator
Consider an operator A and a unitary operator W. Show that A and WAW^{+} have the same eigenvalues.