

PHYS 621: grad Quantum Mechanics I
Due date: Tuesday, September 22, 2026 (in class)

Problem Set #2 Operators & Commutators

Question 1: Spin-1/2 and change of basis

Sakurai & Napolitano problem 1.28

Question 2: Rotations in 2D and Pauli matrices

Sakurai and Napolitano problem 1.5

Question 3: Translation operator and momentum

Sakurai & Napolitano problem 1.32

Question 4: Operator functions

If $f(z)$ is function that can be written with the series expansion $f(z) = \sum_{n=0}^{+\infty} f_n z^n$ with f_n real numbers, then for an operator A we define $f(A) = \sum_{n=0}^{+\infty} f_n A^n$.

a) Consider two operators A and B that commute with their commutator. Show that $[A, f(B)] = [A, B]f'(B)$.

b) Show that $[\vec{R}, f(\vec{P})] = i\hbar \nabla_{\vec{P}} f$ and $[\vec{P}, g(\vec{R})] = -i\hbar \nabla_{\vec{R}} g$, where $\vec{R}$ and $\vec{P}$ are the position and momentum operators.

c) Show the Glauber relation: $e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}$.

Hint: define the function $F(t) = e^{At} e^{Bt}$ and show that it satisfies the differential equation $\frac{dF(t)}{dt} = (A + B + t[A, B])F(t)$. Integrate the equation and use $F(t = 0)$ and $F(t = 1)$.

Question 5: Position and momentum commutator

Calculate $[X^2, P^2]$.

Question 6: Momentum boost operator

Sakurai & Napolitano problem 1.34

Question 7: Momentum and position representations & operators

Sakurai & Napolitano problem 1.36a

Question 8: Dirac delta function

a) Show that $\delta(ax) = \delta(x)/|a|$.

b) Show that $\delta(x^2 - a^2) = [\delta(x - a) + \delta(x + a)]/|2a|$.

Question 9: Gaussian wavepacket

Consider the Gaussian wavepacket $\psi(x) = \frac{1}{\pi^{1/4} d^{1/2}} \exp(-ikx - \frac{x^2}{2d^2})$

Calculate the quantities $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$, and check the uncertainty relation.