

PHYS 621: grad Quantum Mechanics I
Due date: Thursday, October 15, 2026 (in class)

Problem Set #3
Mostly Hamiltonians & Time Evolution

Question 1: Sum of eigenstates

Sakurai & Napolitano problem 1.8

Question 2: Spin-1/2 uncertainty relation

Sakurai & Napolitano problem 1.21

Question 3: Spin-1/2 uncertainty relation ... again

Sakurai & Napolitano problem 1.22

Question 4: 1D Hamiltonian

Sakurai & Napolitano problem 2.6

Question 5: 3D Hamiltonian and virial theorem

Sakurai and Napolitano problem 2.7

Question 6: 2-flavor neutrino oscillations

Sakurai & Napolitano problem 2.4

Question 7: 1D “stationary” Gaussian wavepacket

We start with a Gaussian wavepacket at rest: $\psi(x) = \frac{1}{\pi^{1/4}d^{1/2}} \exp\left(-\frac{x^2}{2d^2}\right)$

(a) Calculate $\langle X \rangle$ and $\langle P \rangle$, as well as $\langle X^2 \rangle$ and $\langle P^2 \rangle$.

(b) Write down the Hamiltonian H for the particle. Is $|\psi\rangle$ an eigenstate of this Hamiltonian? If not, then what are the eigenstates and associated eigenenergies of this Hamiltonian?

(c) Write $|\psi\rangle$ in the eigenbasis of the Hamiltonian.

(d) Calculate the time evolution $|\psi(t)\rangle$ in the eigenbasis of the Hamiltonian.

(e) Write $|\psi(t)\rangle$ in the position space basis, i.e. $\psi(x, t)$, such that $|\psi(t)\rangle = \int_{-\infty}^{+\infty} \langle x|\psi(t)\rangle |x\rangle dx$.

(f) Calculate $\langle X(t) \rangle$ and $\langle P(t) \rangle$, as well as $\langle X(t)^2 \rangle$ and $\langle P(t)^2 \rangle$, and the time evolution of the uncertainty relation $\langle \Delta X(t) \rangle \langle \Delta P(t) \rangle$. How do $\Delta X(t)$ and $\Delta P(t)$ evolve over time?

Question 8: 3-state system with time evolution

We consider the 3-state Hamiltonian in the basis of the three states $\{|1\rangle, |2\rangle, |3\rangle\}$:

$$H = \hbar \begin{bmatrix} 0 & \Omega & 0 \\ \Omega & 0 & \Omega \\ 0 & \Omega & 0 \end{bmatrix}$$

Initially the system is in the $|1\rangle$ state at $t=0$. Derive expressions for the probabilities to be in states $|1\rangle$, $|2\rangle$, and $|3\rangle$ as a function of time, and plot these probabilities versus time (in time units of $1/\Omega$).