PHYS 622: Quantum Mechanics II
Due date: Thursday, January 31, 2013

Problem Set #2

Sakurai and Napolitano problems:
5.1 [5.1], 5.4 [5.4], 5.9 [5.9], 5.11 [5.11], 5.12 [5.12]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

1. Second order perturbation theory correction to the eigenstate
Derive the second order correction to the eigenstate in non-degenerate perturbation theory.

2. Energy shift due to the proton radius in hydrogen
The proton has a radius of $r_p \sim 10^{-15}$ m. In this problem you will use first order perturbation theory to compute the energy shift to the ground state due to the finite size of the proton. *This problem assumes that the proton is a sphere of radius r_p in which the charge is uniformly distributed throughout its volume.*

a) Show that the electric potential energy of the proton and electron is now given by

$$V(r) = -e^2 / r \text{ if } r \geq r_p$$

$$V(r) = \frac{e^2}{2r_p}\left[\left(\frac{r}{r_p}\right)^2 - 3\right] \text{ if } r \leq r_p$$

with $e^2 = q^2/(4\pi\varepsilon_0)$ and q the charge of the electron.

b) Determine the perturbation Hamiltonian term (W) for the standard hydrogen Hamiltonian $H_0 = P^2/(2\mu) - e^2/r$ produced by the modified electric potential energy.

c) Calculate to first order in perturbation energy the correction (i.e. shift) to the ground state energy of hydrogen. For simplicity, you may assume that the radial function varies little over the size of the proton (i.e. $R_{n,l}(r) \approx R_{n,l}(0)$). Give your answer in Hz.

d) Repeat the calculation in the case that the electron is replaced by a point-like muon.