PHYS 622: Quantum Mechanics II
Due date: Friday, April 26, 2013

**Problem Set #9**

**1. Photon Fock states**

Consider the Fock state $|n\rangle_{k,s}$ with $n$ excitations of the photon field with momentum $\boldsymbol{k}$ and polarization $s$ in a volume V (in vacuum).

Compute the following quantities:

a) Average electric field: $\langle \vec{E} \rangle = {}_{k,s}\langle n|\vec{E}|n\rangle_{k,s}$

b) Variance of the electric field: $\Delta\vec{E}^2 = {}_{k,s}\langle n|\vec{E}^2|n\rangle_{k,s} - \left({}_{k,s}\langle n|\vec{E}|n\rangle_{k,s}\right)^2$

c) Average photon number $\langle N \rangle = {}_{k,s}\langle n|N|n\rangle_{k,s}$

and photon number variance $\Delta N^2 = {}_{k,s}\langle n|N^2|n\rangle_{k,s} - \left({}_{k,s}\langle n|N|n\rangle_{k,s}\right)^2$

**2. Photon coherent states**

Consider the coherent state $|\alpha\rangle_{k,s} = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_{k,s}$, where α is a complex number and $|n\rangle_{k,s}$ are the photon Fock states described in problem 1.

Compute the following quantities:

a) Average electric field: $\langle \vec{E} \rangle = {}_{k,s}\langle \alpha|\vec{E}|\alpha\rangle_{k,s}$

b) Variance of the electric field: $\Delta\vec{E}^2 = {}_{k,s}\langle \alpha|\vec{E}^2|\alpha\rangle_{k,s} - \left({}_{k,s}\langle \alpha|\vec{E}|\alpha\rangle_{k,s}\right)^2$

c) Average photon number $\langle N \rangle = {}_{k,s}\langle \alpha|N|\alpha\rangle_{k,s}$

and photon number variance $\Delta N^2 = {}_{k,s}\langle \alpha|N^2|\alpha\rangle_{k,s} - \left({}_{k,s}\langle \alpha|N|\alpha\rangle_{k,s}\right)^2$

**3. Hard sphere scattering**

Consider the 3D spherical potential barrier:

V(r) = 0 for r > a

V(r) = +∞ for r < a

In the limit of s-wave scattering, calculate $\delta_{\ell=0}$, the differential cross-section, and the total cross-section.

**4. Spherical well scattering**

Consider the 3D spherical potential well:

V(r) = 0 for r > a

V(r) = $-V_0$ for r < a

a) Calculate $\delta_{\ell=0}$.

b) Calculate $\delta_{\ell=1}$.

You are encouraged to consult Appendix B.5 of Sakurai and Napolitano [Appendix A.5 of Sakurai (red)].

**5. Ramsauer-Townsend effect**

Consider the 3D spherical well of problem #4. In the limit $k \to 0$, expand $\delta_{\ell=0}$ in powers of $k$:

$$\delta_{\ell=0} = \alpha_1 k + \alpha_2 k^2 + \dots$$

a) Evaluate the first coefficient $\alpha_1$. What is the value of $V_0 a^2$ for which $\alpha_1=0$ ?

At this value there is no s-wave scattering for $k \to 0$: this is the Ramsauer-Townsend effect.

b) In the case of $\alpha_1=0$, plot the s-state wave function $u_0$ inside the potential. Plot the tangent line at $r = a$ and show that it goes through the origin.

Note: the result of problem #4 a) is

$$\tan\delta_{\ell=0} = \frac{k\tan(Ka) - K\tan(ka)}{K + k\tan(Ka)\tan(ka)}, \text{ where } K = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \text{ and } k = \sqrt{\frac{2mE}{\hbar^2}}.$$