DC Zeeman Effect

Case 1: weak field limit $\mu B \ll A$

- We treat $W = H_{Zeeman}$ as a perturbation to $H_0 + H_{SO} + H_{HF}$
- We work in the $\{|\psi\rangle = |n\rangle_\ell |F, m_F\rangle\}$ basis

↳ F^2 and F_z are good quantum numbers

$$H = H_0 + H_{SO} + H_{HF} + \underbrace{\left\{\left(\frac{-q_e}{2m_e}\right)(\vec{L} + 2\vec{S})\cdot\vec{B} - \frac{g_I |q_e|}{2m_p}\vec{I}\cdot\vec{B}\right\}}_{H_{Zeeman}}$$

we found that $H_{Zeeman}\Big|_{F\text{-subspace}} \simeq g_F \mu_B \vec{F}\cdot\vec{B} = g_F \mu_B |B| m_F$

$\vec{B} = |B|\hat{z}$ — linear Energy shift

g_F = Landé g-factor

Case 2: strong field limit $\mu B \gg A$

We treat $W = H_{HF}$ as a perturbation to $H_0 + H_{SO} + H_{Zeeman}$

$$H_0 + H_{SO} + H_{Zeeman} = H_0 + \frac{1}{2}\frac{e^2}{m_e^2 c^2}\frac{1}{R^3}\vec{L}\cdot\vec{S} + \left[\left(\frac{-q_e}{2m_e}\right)(\vec{L}+2\vec{S}) - \frac{g_I|q_e|}{2m_p}\vec{I}\right]\cdot\vec{B}$$

$\vec{L}\cdot\vec{S} \to \frac{1}{2}(J^2 - L^2 - S^2)$, $L_z + 2S_z = J_z + S_z$, $\vec{I} \to I_z$

good quantum numbers: $J^2, J_z, L^2, S^2, S_z, I_z$

i.e. ~~good~~ diagonalizing basis: $\{|\psi\rangle = |n\rangle_\ell |J, m_J\rangle |I, m_I\rangle\}$

what about $S_z|\psi\rangle = ?$ or $S_z|J, m_J\rangle = ?$

within a J-subspace $\langle J, m_J'|S_z|J, m_J\rangle = 0$ for $m_j' \neq m_j$

because $|J, m_j\rangle = \alpha\underbrace{|m_\ell = m_j + \tfrac{1}{2}\rangle|m_s = -\tfrac{1}{2}\rangle}_{m_\ell + m_s = m_j} + \beta\underbrace{|m_\ell = m_j - \tfrac{1}{2}\rangle|m_s = +\tfrac{1}{2}\rangle}_{m_\ell + m_s = m_j}$

these two basis vectors cannot show up in $|J, m_j'\rangle$ if $m_j' \neq m_j$

$\hookrightarrow$ S_z does not change basis vectors, only the coefficients

using the Wigner-Eckart projection theorem:

$$\langle J, m_j|\vec{S}|J, m_J\rangle = \underbrace{\frac{\langle J, m_j|\overbrace{\vec{J}\cdot\vec{S}}^{\frac{1}{2}(J^2+S^2-L^2)}|J, m_j\rangle}{\langle J, m_j|J^2|J, m_j\rangle}}_{\pm\frac{1}{2\ell+1} \text{ for } J = \ell \pm \frac{1}{2}} \langle J, m_j|\vec{J}|J, m_j\rangle$$

$$\langle J, m_j|S_z|J, m_j\rangle = \pm\frac{m_j\hbar}{2\ell+1} \quad \text{for } j = \ell \pm 1/2$$

We apply perturbation theory (1st order) to

$$W = H_{HF} = \frac{A}{\hbar^2}\vec{I}\cdot\vec{J} = \frac{A}{\hbar^2}\frac{1}{2}\left(\vec{F}^2 - \vec{J}^2 - \vec{I}^2\right)$$

this is not helpful since we do not know how to evaluate

$\langle I, m_I|\langle J, m_j|\sum_\ell \langle n|F^2|n\rangle|J, m_j\rangle|I, m_I\rangle$

$$= \frac{A}{\hbar^2}\left(I_x J_x + I_y J_y + I_z J_z\right)$$

$$= \frac{A}{\hbar^2}\left[\frac{1}{2}\left(J_+ I_- + J_- I_+\right) + J_z I_z\right]$$

↳ use this form to compute $\langle\psi| \frac{A}{\hbar^2} \vec{I}\cdot\vec{J} |\psi\rangle$

<u>note</u>: We do not need to compute 0^{th} order correction to eigenstates since there is no degeneracy when $|B| \neq 0$.

<u>Example</u>: 1s level of Hydrogen $\quad \vec{J} = \vec{S}$

$I = 1/2$ (proton), $\quad S = 1/2$ (e^-)

$$\langle m_I m_s | \frac{A}{\hbar^2} \vec{I}\cdot\vec{J} | m_I m_s\rangle = \langle m_I m_s | \frac{A}{\hbar^2}\left[\frac{\overset{0}{\cancel{S_+ I_-}} + \overset{0}{\cancel{S_- I_+}}}{2} + S_z I_z\right] | m_I m_s\rangle$$

$$= A\, m_I m_s$$

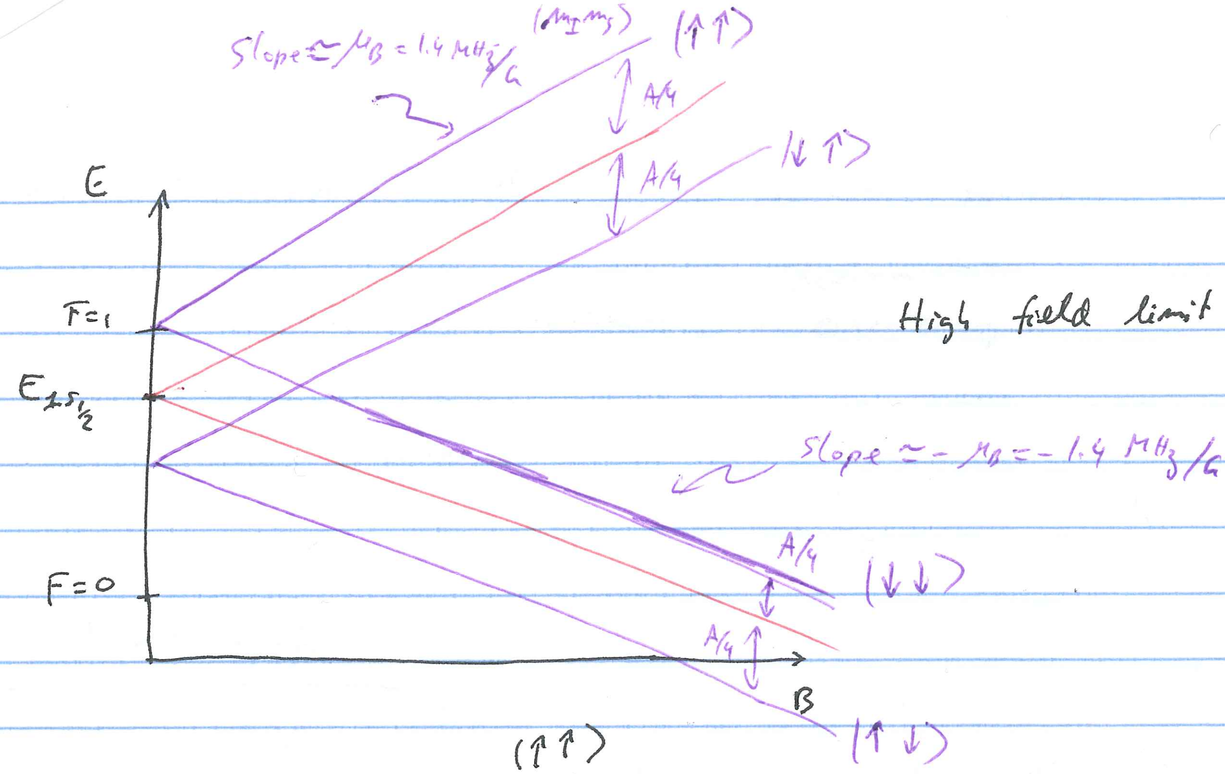
However, we must also compute

$$\langle m_I m_s | \overset{E_{1s_{1/2}}}{H_0} + \overset{0\ (l=0)}{H_{SO}} + H_{Zeeman} | m_I m_s\rangle$$

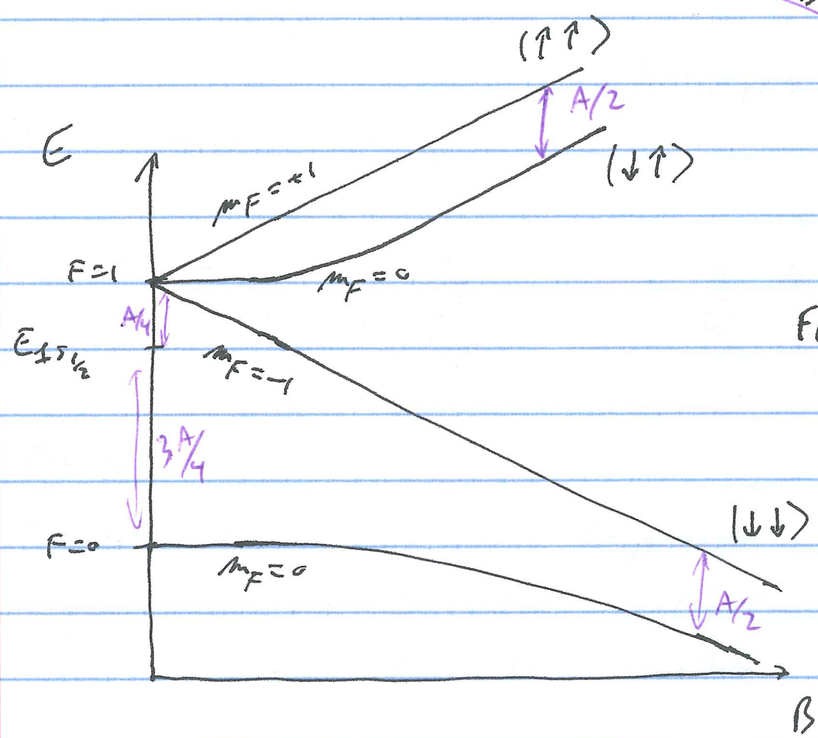
$$= E_{1s_{1/2}} + \langle m_I m_s | \left(-\frac{q_e}{m_e} \overset{\hbar m_s}{S_z} - \frac{g_I |q_e|}{2 m_p} \overset{\hbar m_I}{I_z}\right)|B|\, | m_I m_s\rangle$$

$$= E_{1s_{1/2}} + 2\mu_B m_s |B| - g_I \mu_N m_I |B|$$

thus $\quad E_{total} = E_{1s_{1/2}} + \left(2\mu_B m_s - g_I \mu_N m_I\right)|B| + \underbrace{A\, m_I m_s}_{\pm\frac{A}{4}}$



High field limit



Full calculation

~~t=0~~

Case 3: Intermediate field $\quad \mu B \sim A$

$\Rightarrow$ diagonalize $\quad H_{HF} + H_{zeeman}$ in $\{|F, m_F\rangle\}$ basis

↳ generally done numerically

or

$\{|I, m_I\rangle|S, m_s\rangle\}$ basis

but in the given J subspace

note: you don't actually need perturbation theory!

for $nS_{1/2}$ levels there is an analytic formula:

Breit-Rabi formula, for Zeeman shift eigenenergies

[Show overheads]

Stark Effect

$(\vec{B}=0, \vec{A}=0)$. We apply a uniform electric field $\vec{E}=|E|\hat{z}$.
The Hamiltonian is

$$H = H_0 + \frac{1}{2}\frac{e^2}{m_e^2 c^2}\frac{1}{R^3}\vec{L}\cdot\vec{S} + \frac{A}{\hbar^2}\vec{I}\cdot\vec{J} \underbrace{- q_e \vec{E}\cdot\vec{R}}$$

$H_{stark} = -q_e|E|Z$ for $\vec{E}=|E|\hat{z}$

Perturbation theory

1- If different L-states are degenerate, then H_{stark} will lift degeneracy $\rightarrow$ 1st order correction to Energy $\rightarrow$ Linear stark effect
0th order correction to Eigenstates

2- If there is no L degeneracy, then H_{stark} will not lift $|F, m_F\rangle$ degeneracy.
$\rightarrow$ no 1st order correction to Energy
$\rightarrow$ { 2nd order correction to Energy $\rightarrow$ quadratic Stark effect
1st order correction to eigenstates
↳ Stark effect mixes states of opposite parity (even & odd L-states)