PHYS 622: Quantum Mechanics II
Due date: Thursday, February 21, 2013

Problem Set #3

Sakurai and Napolitano problems:
5.7 [5.7], 5.10 [5.10], 5.15 [5.15], 5.16 [5.16], 5.18 [5.18]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

1. The $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen

Consider the following Hamiltonian for the hydrogen atom

$$H = H_0 - \frac{P^4}{8m_e^3c^2} + \frac{1}{2}\frac{e^2}{m_e^2c^2}\frac{1}{R^3}\vec{L}\cdot\vec{S} \quad \text{with } H_0 = \frac{P^2}{2m_e} - \frac{e^2}{R}$$

(We will not consider the Lamb shift in this problem)

a. Determine the 0th order correction to the eigenstates (i.e. identify a suitable basis which diagonalizes this Hamiltonian) starting from the eigenstates of H_0.

b. Compute the 1st order correction to the eigen-energies of H_0 corresponding to the $2S_{1/2}$ and $2P_{1/2}$ states.

c. We now add an electric field E directed along the z-axis. Compute the Stark shifts of the n=2, J=1/2 levels to lowest non-zero order using perturbation theory.

2. Zeeman splittings of an *nS* level for an alkali atom at arbitrary magnetic field

Calculate and plot the energies (relative to the hyperfine splitting) of all the states to first order of an $nS_{1/2}$ level of an alkali atom (such as ^{6}Li) with $I=1$, $S=1/2$, and a hyperfine splitting of ΔE_{HF} at arbitrary magnetic field. Also, calculate the hyperfine A coefficient. DO NOT use the Breit-Rabi formula or perturbation theory.