PHYS 622: Quantum Mechanics II
Due date: Thursday, April 4, 2013

Problem Set #7

Sakurai and Napolitano problems
4.2 [4.2], 4.3 [4.3], 4.10* [4.10], 4.11 [4.11], 4.12 [4.12]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

* Sakurai and Napolitano problem 4.10 revised

(a) Use (4.4.53) to show that $\Theta|jm\rangle$ equals $|j, -m\rangle$ up to some phase that includes the factor $(-1)^m$. That is, show that $\Theta|jm\rangle = e^{i\delta}(-1)^m|j, -m\rangle$, where δ is independent of m.

(b) Using the same phase convention, find the time-reversed state corresponding to $\mathcal{D}(R)|jm\rangle$. Consider using the infinitesimal form $\mathcal{D}(\hat{n}, d\phi)$ and then generalize to finite rotations.

(c) From these results, prove that, independent of δ, one finds

$$\mathcal{D}_{m'm}^{(j)*}(R) = (-1)^{m-m'}\mathcal{D}_{-m',-m}^{(j)}(R)$$

(d) Conclude that we are free to choose $\delta = 0$, and $\Theta|jm\rangle = (-1)^m|j, -m\rangle = i^{2m}|j, -m\rangle$.

Note: Equation 4.4.53 is the same in Sakurai and Napolitano and in the old (red) Sakurai (revised, 1st ed.)