PHYS 622: Quantum Mechanics II
Due date: Friday, April 18, 2014

Problem Set #9

1. Hard sphere scattering

Consider the 3D spherical potential barrier:

$V(r) = 0$ for $r > a$

$V(r) = +\infty$ for $r < a$

In the limit of s-wave scattering, calculate $\delta_{\ell=0}$, the differential cross-section, and the total cross-section.

2. Spherical well scattering

Consider the 3D spherical potential well:

$V(r) = 0$ for $r > a$

$V(r) = -V_0$ for $r < a$

a) Calculate $\delta_{\ell=0}$.

b) Calculate $\delta_{\ell=1}$.

You are encouraged to consult Appendix B.5 of Sakurai and Napolitano [Appendix A.5 of Sakurai (red)].

3. Ramsauer-Townsend effect

Consider the 3D spherical well of problem #4. In the limit $k \to 0$, expand $\delta_{\ell=0}$ in powers of k:

$$\delta_{\ell=0} = \alpha_1 k + \alpha_2 k^2 + \ldots$$

a) Evaluate the first coefficient α_1. What is the value of $V_0 a^2$ for which $\alpha_1 = 0$?

At this value there is no s-wave scattering for $k \to 0$: this is the Ramsauer-Townsend effect.

b) In the case of $\alpha_1 = 0$, plot the s-state wave function u_0 inside the potential. Plot the tangent line at $r = a$ and show that it goes through the origin.

Note: the result of problem #2 a) is

$$\tan\delta_{\ell=0} = \frac{k\tan(Ka) - K\tan(ka)}{K + k\tan(Ka)\tan(ka)}, \text{ where } K = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \text{ and } k = \sqrt{\frac{2mE}{\hbar^2}}.$$